The Labor Wedge:
A Search and Matching Perspective

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Abstract

We define and quantify static and dynamic labor market wedges in a search and matching model with endogenous labor force participation. The dynamic labor wedge is a novel object that is not present in Walrasian frameworks due to the absence of long-lasting work relationships. We find that, in a version of the model where all employment relationships turn over every period, the (static) labor wedge is countercyclical, a result that is consistent with existing literature. Once we consider long-lasting employment relationships, we can measure both static and dynamic wedges separately. We then find that, while the static wedge continues to be countercyclical, the dynamic (or intertemporal) wedge is procyclical. The latter suggests that understanding the behavior of labor demand may be crucial to understand the dynamic wedge. One possible rationale behind the behavior of the dynamic wedge is the "cleansing" effects of recessions.

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1 Introduction

This paper measures labor wedges using a labor search-and-matching framework. The key innovation in measuring ine ciencies in our environment lies in exploiting both the extensive margin of employment and the presence of endogenous labor force participation together with the fact that work relationships are long-lasting in order to construct model-consistent wedges. The presence of

The second step consists of allowing for long-lasting jobs, which the search and matching literature naturally describes. The long-lasting nature of employment relationships introduces a second, *intertemporal* an environment with frictions in the goods market and finds that households' search behavior for goods appears as a labor wedge that resembles a countercyclical labor income tax. However, he does not consider the presence of dynamic inefficiencies as a result of long-lasting relationships in the goods market.[3] Similar to Duras (2015b), Bils, Malin, and Klenow (2014) argue that movements in the product market wedge | reflected in price markups that arise from a richer production function

Notation	Description	Notes
c_t	Consumption in period t	
s_t	Search activity in period t	
v_t	Vacancies posted in period t	
n_t	Employment in period t	
lfp_t	Labor-force participation in period t	$(1\ \ )n_{t\ \ 1} + s_t$
$_t$	Labor-market tightness	v_t/s_t
p_t	Job- nding probability	Depends on $_t$ if CRS matching

Table 1: Notation.

2 Theoretical Framework

The model uses the \instantaneous hiring" view of transitions between search unemployment and employment, in which new hires begin working right away, rather than with a one-period delay (see Arseneau and Chugh, 2012). Basic notation of the model is presented in Table 1, and Figure 1 summarizes the timing of the model. At the beginning of any period t, a fraction of employment relationships that were active in period t 1 exogenously separates. Some of these newly-separated individuals may immediately enter the period-t job-search process, as may some individuals who were non-participants in the labor market in period t 1; these two groups taken together constitute the measure s_t of individuals searching for jobs in period t.

A constant-returns-to-scale aggregate matching function randomly assigns some fraction of these s_t individuals to job matches. More precisely, of these s_t individuals, $(1\ \ p_t)s_t$ individuals turn t out to be unsuccessful in their job searches, where

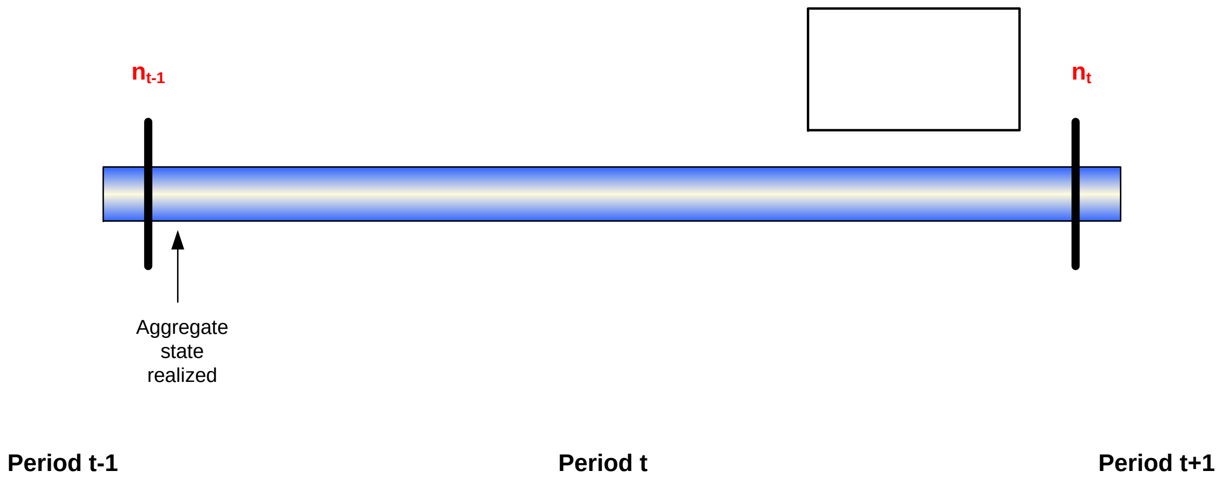


Figure 1: Timing of events.

subject to a sequence of aggregate resource constraints

$$c_t + \gamma(v_t) + g_t = z_t f(n_t), \tag{2}$$

and a sequence of aggregate laws of motion for employment

$$n_t = (1 - \rho)n_{t-1} + m(s_t, v_t). \tag{3}$$

The total vacancy posting costs in (2) may or may not be linear in v_t, depending on the functional form of the function $\gamma(\cdot)$. Also note that the argument in the subutility function $h(\cdot)$ is labor-force participation; in turn, because efficient allocations take account of possible congestion externalities, p_t depends on aggregate labor-market tightness θ_t.

Efficient allocations $\{c_t, lfp_t, v_t, n_t\}_{t=0}^{\infty}$ are characterized by the sequence of labor-force participation conditions

$$\frac{h'(lfp_t)}{u'(c_t)} = \gamma'(v_t)\frac{m_s(s_t, v_t)}{m_v(s_t, v_t)}, \tag{4}$$

job-creation conditions

$$\frac{\gamma'(v_t)}{m_v(s_t, v_t)} - z_t f'(n_t) = (1 - \rho)E_t\left\{\beta\frac{u'(c_{t+1})}{u'(c_t)}\frac{\gamma'(v_{t+1})}{m_v(s_{t+1}, v_{t+1})}\left[1 - m_s(s_{t+1}, v_{t+1})\right]\right\}, \tag{5}$$

and the sequence of technological frontiers described by (2) and (3). In the efficient labor-force participation condition (4) and the efficient job-creation condition (5), the marginal products of the matching function, $m_v(\cdot)$ and $m_s(\cdot)$, appear because they are components of the technological frontier of the economy. The formal analysis of this problem appears in Appendix A.

2.2 "Zero Wedges"

To highlight the "zero-wedges" view, it is useful to restate efficiency in terms of MRSs and corresponding MRTs. For the intertemporal condition, this restatement is most straightforward for the non-stochastic case, which allows an informative disentangling of the preference and technology terms inside the $E_t(.)$ operator in (5).

Proposition 1. *Efficient Allocations. The MRS and MRT for the pairs* (c_t, lfp_t) *and* (c_t, c_{t+1}) *are defined by*

$$MRS_{c_t,lfp_t} \equiv \frac{h'(lfp_t)}{u'(c_t)} \qquad MRT_{c_t,lfp_t} \equiv \gamma'(v_t)\frac{m_s(s_t,v_t)}{m_v(s_t,v_t)}$$

$$IMRS_{c_t,c_{t+1}} \equiv \frac{u'(c_t)}{u'(c_{t+1})} \qquad IMRT_{c_t,c_{t+1}} \equiv \frac{(1-\rho)\left[\frac{\gamma'(v_{t+1})}{m_v(s_{t+1},v_{t+1})}\left[1-m_s(s_{t+1},v_{t+1})\right]\right]}{\frac{\gamma'(v_t)}{m_v(s_t,v_t)}-z_t f'(n_t)}. \tag{6}$$

Static efficiency (4) is characterized by $MRS_{c_t,lfp_t} = MRT_{c_t,lfp_t}$*, and (for the non-stochastic case) intertemporal efficiency is characterized by* $IMRS_{c_t,c_{t+1}} = IMRT_{c_t,c_{t+1}}$*.*

Proof. See Appendix A. □

As described in Arseneau and Chugh (2012), each MRS in Proposition 1 has the standard interpretation as a ratio of relevant marginal utilities. By analogy, each MRT has the interpretation as a ratio of the marginal products of an appropriately-defined transformation frontier.[5] Efficient allocations are then characterized by an MRS = MRT condition along each optimization margin, implying zero distortions on each margin. However, rather than taking the efficiency conditions as prima facie justification that the expressions in Proposition 1 *are* properly to be understood as MRTs, each can be described conceptually from first principles, independent of the characterization of efficiency. Formal details of the following mostly intuitive discussion appear in Appendix A.

2.2.1 Static MRT

To understand the static MRT in Proposition 1, MRT_{c_t,lfp_t}, consider how the economy can transform a unit of non-participation (leisure) in period t into a unit of consumption in period t, holding

[5] We have in mind a very general notion of transformation frontier as in Mas-Colell, Whinston, and Green (1995, p. 129), in which every object in the economy can be viewed as either an input to or an output of the technology to

output constant. A unit reduction in leisure allows a unit increase in s_t, which in turn leads to $m_s(s_t; v_t)$ new employment matches in period t. Each of these new matches, in principle, produces $z_t f'(n_t)$ units of output, and hence consumption. The overall marginal transformation between leisure and consumption described thus far is $z_t f'(n_t) m_s(s_t; v_t)$.

However, in order to hold output constant in this transformation, the number of vacancies must tertemporal e ciency condition can thus be represented as

$$1 = E_t \left\{ u'(c_{t+1}) \right.$$

deviation of IMRS from IMRT:

$$1 = E_t \left\{ \frac{1}{\tau^D_t} \frac{u'(c_{t+1})}{u'(c_t)} \left[\frac{\frac{\beta(1-\delta)\kappa'(v_{t+1})(1-m_s(s_{t+1};v_{t+1}))}{m_v(s_{t+1};v_{t+1})}}{\frac{\kappa'(v_t)}{m_v(s_t;v_t)} - z_t f'(n_t)} \right] \right\} : \qquad (10)$$

In the same way that the static wedge is associated to the supply side of the labor market, the dynamic wedge is associated to the demand side.

We quantify the static and dynamic wedges in a search and matching framework as defined previously, and build hypotheses on what could be the forces behind its secular evolution as well as its business cycle fluctuations. Unlike the Walrasian framework that only features the static wedge, the search and matching model introduces the dynamic dimension through long-lasting

Functional Form	Description
$u(c_t) = \ln c_t$	Consumption subutility
$h(x) = \frac{}{1+1/\varepsilon} x^{1+1/\varepsilon}$	Labor force participation subutility
$m(s_t; v_t) = \ {}_t s_t v_t^{1}$	Aggregate matching technology
$f(n_t) = z_t n_t$	Goods-production technology
$(v_t) = \left[\ + \ (v_t \ \ v)^2 \right] v_t$	Vacancy posting cost

Table 2: Functional forms.

Parameter	Baseline Value	Description
		Utility parameters
	0.99	Household's subjective discount factor
ε	0.18	Frisch elasticity
		Technology parameters
	0.7	Elasticity of goods production wrt n
	0.5	Elasticity of matching wrt s
	0	Convexity of vacancy posting costs
	= 1 (full turnover)	Job separation rate
	< 1 (dynamic model)	

Table 3: Parameter Values.

3.1 Parameterization and Functional Forms

We choose standard functional forms for preferences as well as for the production and matching technologies, as shown in Table 2. The vacancy posting cost function is chosen so that the steady state is not affected by the degree of convexity, which is useful for comparison purposes.

Regarding the calibration of the parameters, first note that in our framework the notion of labor supply is along the extensive margin. More precisely, it is the elasticity of *labor force participation* that the parameter ε captures, rather than the elasticity of hours worked. Following Arseneau and Chugh (2012), we initially set this elasticity at $\varepsilon = 0.18$, but we consider a range of other parameter settings for ε in Section 5. The preference and production parameters are standard in business cycle models. For reference, Table 3 displays the baseline parameter values.

3.2 Data

Measuring the labor wedges for frictional labor markets, as defined in Section 2.4, requires data on five series: the employment rate, the labor force participation rate, the consumption and government shares, and the vacancy rate. The analysis is done at a quarterly frequency, and the period considered is 1951$Q1$ to 2013$Q4$. Appendix B presents our main results for the period 1980Q1 to 2013Q4 for robustness.

Private consumption, government spending and output are measured using Real Personal Consumption Expenditures, Real Government Consumption Expenditures and Gross Investment, and Real Gross Domestic Product, respectively. Seasonally-adjusted data for these variables in chained 2009 dollars are obtained from the Bureau of Economic Analysis (BEA) (NIPA Table 1.5.6).

Since our baseline model abstracts from the intensive margin, the wedges are defined along the extensive margin, requiring data on the employment rate rather than on hours worked. The source of employment data is the Bureau of Labor Statistics (BLS). The variables n_t and lfp_t are measured using the Civilian Employment-Population Ratio and the Civilian Labor Force Participation Rate (series LNS12300000 and LNS11300000), respectively. Both series are seasonally-adjusted.

Finally, the vacancy rate is defined as the number of vacancies (job openings) divided by the sum of total payroll employment plus the number of vacancies. The series for vacancies corresponds to the seasonally adjusted level of vacancies from JOLTS (Job Openings and Labor Turnover Survey) for the period 2000-2013, which we combine with the Composite Help-Wanted Index constructed by Barnichon (2010) to extend the vacancy series back to 1951. A scaling factor is used to ensure the level of vacancies computed using the composite index matches the level observed in December 2000 in JOLTS. The series of total payroll employment in the non-farm sector is obtained from BLS (series CES0000000001).

4 Results

4.1 Short-Run Relationships

which, given the functional forms considered, can be rewritten as

$$\frac{lfp_t^{1/\varepsilon}}{1/c_t} = \frac{y_t}{n_t} \quad {}_t \quad \frac{s_t}{v_t} \quad {}^{1} : \tag{12}$$

Recall that with full turnover, $s_t = lfp_t$. In addition, from the law of motion for n_t:

$$\frac{n_t}{lfp_t} = {}_t \quad \frac{lfp_t}{v_t} \quad {}^{1} : \tag{13}$$

Then, the labor market wedge when = 1 is given by

$$= \; — \; \frac{1}{c_t/y_t} \; lfp_t^{\;(1+1/\varepsilon)} : \tag{14}$$

Equation (14) is used to compute the empirical measure of the labor market wedge for the full turnover case, based on the data and parameterization described in Section 3. The resulting wedge, log-linearized around its steady state, is shown in Figure 2.[9]

At low frequencies, Figure 2 shows a substantial reduction in the labor market wedge between the 1960s and the 1990s. It remained relatively at throughout the 1990s and the early 2000s. Since 2008, the wedge has been on the rise, and exhibited a notable increase after the Great Recession. The wedge displays this general pattern even for di erent parameterizations of the elasticity of labor participation, ε, as discussed in Section 5. Given a small elasticity of participation, the observed behavior in the wedge mirrors the evolution of labor force participation over the last 50 years (Figure

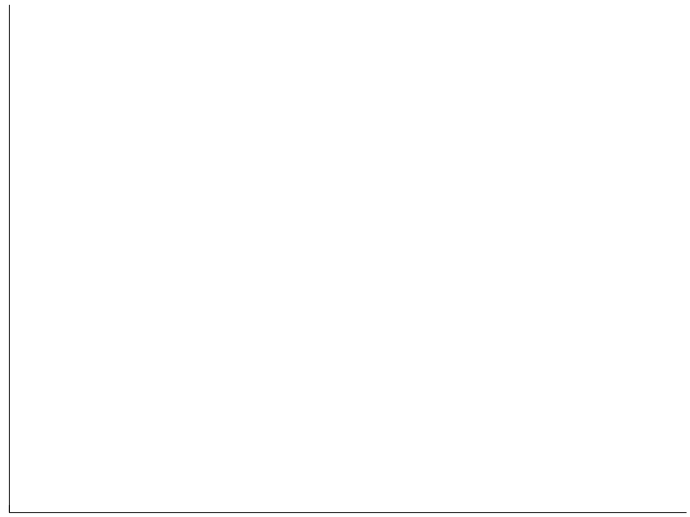


Figure 2: Labor Market Wedge in Frictional Labor Markets. Short-run relationships (= 1). Shaded areas indicate NBER recessionary periods.

additional inefficiencies. Another potential (and related) explanation is related to the increase in schooling: the fact that young individuals continue their studies rather than starting to work implies that more and more participants have college degrees, making the screening by employers and the processing of information regarding job candidates more difficult. Additional screening efforts divert resources from productive uses, thus leading to larger inefficiencies. All these plausible explanations may be relevant for understanding the changes in the labor wedge in the data but their plausibility

n the tu

	Std. Dev.	Relative Std. Dev.	1st Order Autocorrel.	Correl. w/ Output
		= 1		
c/y	0.005	0.614	0.587	0.101
g/y	0.009	1.142	0.874	0.603
n	0.006	0.808	0.871	0.162
lfp	0.003	0.350	0.511	0.114
v	0.004	0.542	0.895	0.313
	0.082	10.251	0.419	-0.337
		= 0.66		
c/y	0.005	0.445	0.587	-0.013
g/y	0.009	0.828	0.874	0.425
n	0.006	0.586	0.871	0.362
lfp	0.003	0.254	0.511	0.209
v	0.004	0.393	0.895	0.539
^{S}	0.084	7.634	0.343	-0.330
^{D}	0.172	15.580	0.780	0.368
		= 0.25		
c/y	0.005	0.146	0.587	-0.236
g/y	0.009	0.272	0.874	-0.010
n	0.006	0.193	0.871	0.691
lfp	0.003	0.084	0.511	0.356
v	0.004	0.129	0.895	0.885
^{S}	0.114	3.389	0.321	0.063
^{D}	0.125	3.723	0.534	0.249

Table 4: Business Cycle Statistics, United States, 1951Q1 - 2013Q4. Cyclical components are computed using HP lter with = 1600. denotes the labor wedge for the full turnover case (= 1). $^{S}_{n}$ and $^{D}_{n}$ are the \static" and \dynamic" labor market wedges. All wedges are computed assuming a Frisch elasticity of 0.18 and linear vacancy posting costs.

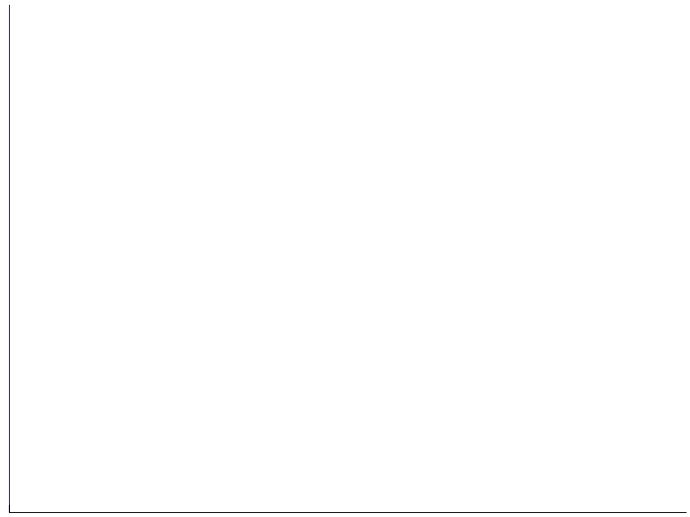


Figure 3: The Link between the Labor Market Wedge and the Labor Force Participation.

Given the functional forms considered, the wedges in equations (9) and (10) can be rewritten as

$$ ^{s} = \frac{}{(1 \quad)} \frac{\left[v_t \quad + \quad (v_t \quad v)^2 + 2 \; v_t \, (v_t \quad v) \right]}{c_t} \; lfp $$

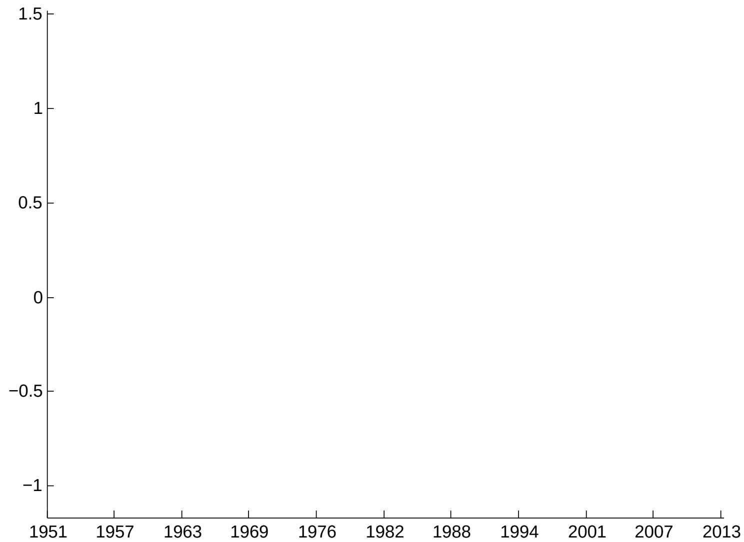


Figure 4: Static Wedge in Frictional Labor Markets. Shaded areas indicate NBER recessionary periods.

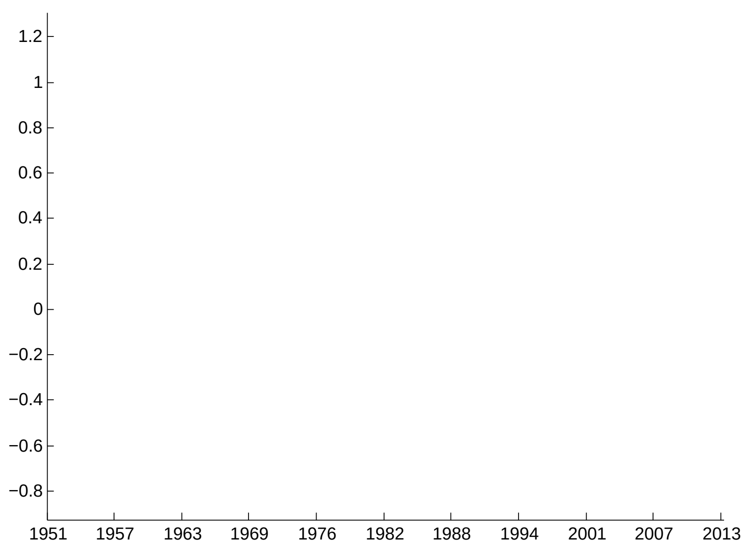


Figure 5: Dynamic Wedge in Frictional Labor Markets. Shaded areas indicate NBER recessionary periods.

the demand side of the market, related to the vacancy-posting decision of the firms. This stands in contrast to the emphasis of in the Walrasian-based literature that emphasizes the (household) supply side (Karabarbounis, 2014, and others).

Figure 5 shows that the dynamic wedge has declined steadily since the 1960s although less so relative to the static wedge. This could be associated to the introduction of new technologies, which have made vacancy posting easier and cheaper. An alternative explanation is the increased substitution of labor by capital[11]; the reallocation towards physical capital may allow firms to more effectively reduce inefficient vacancy postings.

Regarding the cyclical fluctuations in the dynamic wedge, when looking at the contemporaneous

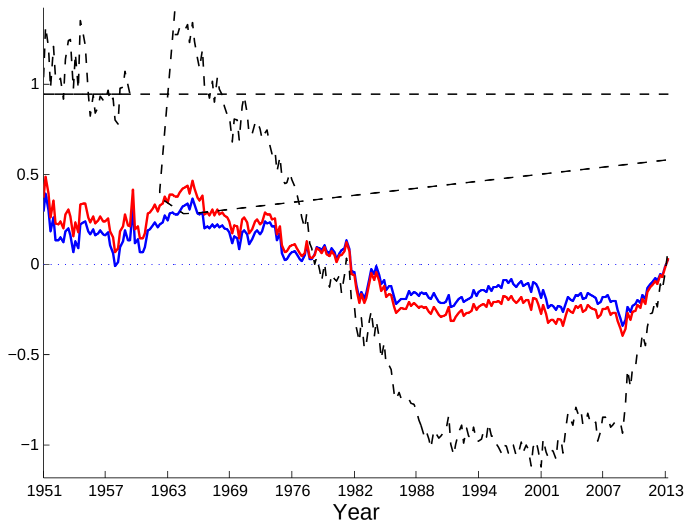
	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}

= 0:66

The intuition is simple: in the presence of convex vacancy posting costs, vacancies will respond more aggressively to large positive and negative shocks (technological wedges), thereby making the dynamic wedge more volatile. However, since the functional form of the convexity of vacancy postings we use does not a ect the steady state, the trend remains e ectively identical to the speci cation with linear vacancy posting costs.

6 Conclusions

(a) Static Wedge



(b) Dynamic Wedge

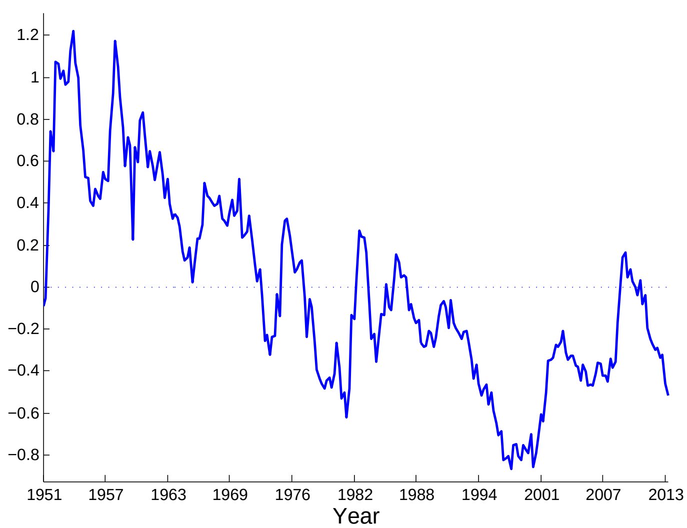
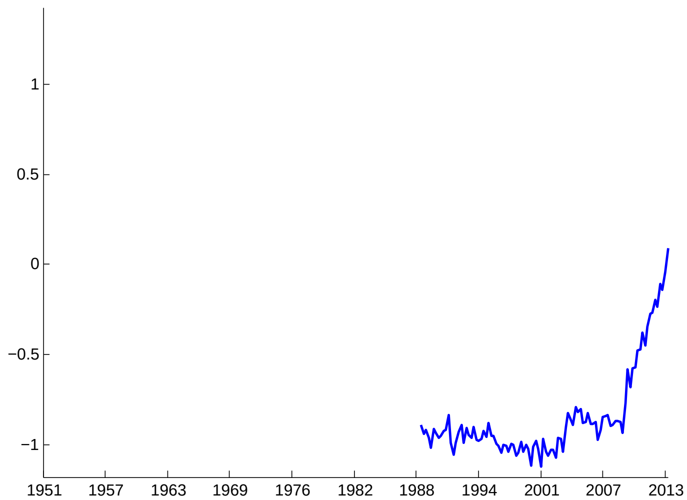


Figure 6: Sensitivity to Different Values of the Elasticity of Labor Force Participation. Shaded areas indicate NBER recessionary periods. Both the static and dynamic wedges are computed for = 0.66.

(a) Static Wedge



(b) Dynamic Wedge

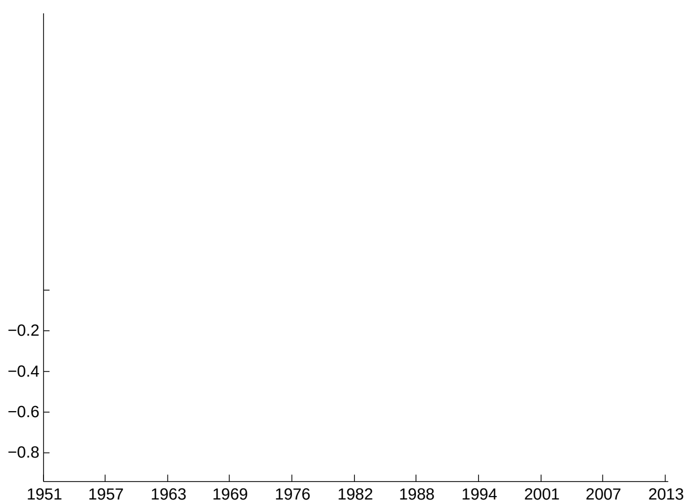


Figure 7: Sensitivity to Di erent Degrees of Convexity of the Vacancy Posting Cost Function. Shaded areas indicate NBER recessionary periods. Both the static and dynamic wedges are computed for = 0:66.

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For Cobb-Douglas matching and its associated marginals,[14] static e ciency is characterized by

$$\frac{h'(lfp_t)}{u'(c_t)} = \;'(v_t)\frac{\quad}{1}\;\;_t. \tag{25}$$

Because its derivation relies only on the static rst-order conditions (20), (21), and (22), we interpret (24) (or (25)) as the model's static e ciency condition.

A.2 Intertemporal E ciency

Using conditions (20) and (21) to eliminate the multipliers from (23) gives

$$\frac{'(v_t)}{m_v(s_t; v_t)} = z_t f'(n_t$$

Consider the static efficiency condition (24). The left-hand side is clearly the within-period MRS between consumption and participation (search) in any period t. We claim that the right-hand side is the corresponding MRT between consumption and participation. Rather than taking the efficiency condition (24) as prima facie evidence that the right-hand side *must be* the static MRT, however, this MRT can be derived from the primitives of the environment (i.e., independent of the context of any optimization).

First, though, we define MRS and MRT relevant for intertemporal efficiency. To do so, we first restrict attention to the non-stochastic case because it makes clearer the separation of components of preferences from components of technology (due to endogenous covariance terms implied by the $E_t(.)$ operator). The non-stochastic intertemporal efficiency condition can be expressed as

$$\frac{u'(c_t)}{u'(c_{t+1})}$$

gives

$$n_t - (1-\rho)n_{t-1} - m\left(s_t; \kappa^{-1}(z_t f(n_t) - c_t)\right) = 0. \tag{30}$$

Next, use the accounting identity $lfp_t = (1-\rho)n_{t-1} + s_t$ to substitute for s_t on the right-hand side, and define

$$\Phi(c_t; lfp_t; n_t; .) \equiv n_t - (1-\rho)n_{t-1} - m\left(lfp_t - (1-\rho)n_{t-1}; \kappa^{-1}(z_t f(n_t) - c_t)\right) = 0 \tag{31}$$

as the period-t transformation frontier. The function $\Phi(.)$ is a more general notion of a transformation, or resource, frontier than either the goods resource constraint or the law of motion for employment alone because $\Phi(.)$ *jointly* describes the *two* technologies in the economy: the technology that creates employment matches and, conditional on employment matches, the technology that creates output. The dependence of $\Phi(.)$ on (among other arguments) c_t and lfp_t is highlighted because the period-t utility function is defined over c_t and lfp_t.

c_{t+1} and c_t

$$G_{c_{t+1}} = m_v(s_{t+1}; v$$

B Business Cycle Statistics 1980-2013

	Std. Dev.	Relative Std. Dev.	1st Order Autocorrel.	Correl. w/ Output
= 1				
c/y	0.004	0.815	0.628	0.653
g/y	0.004	0.833	0.867	0.199
n	0.006	1.195	0.904	0.043
lfp	0.002	0.395	0.513	0.063
v	0.003	0.671	0.893	0.292
	0.061	11.952	0.457	-0.327
= 0:66				
c/y	0.004	0.574	0.628	0.434
g/y	0.004	0.586	0.867	-0.076
n	0.006	0.842	0.904	0.296
lfp	0.002	0.278	0.513	0.141
v	0.003	0.473	0.893	0.567
^{S}	0.064	8.781	0.405	-0.232
^{D}	0.134	18.441	0.787	0.188
= 0:25				
c/y	0.004	0.169	0.628	-0.042
g/y	0.004	0.173	0.867	-0.536
n	0.006	0.248	0.904	0.666
lfp	0.002	0.082	0.513	0.246
v	0.003	0.139	0.893	0.913
^{S}	0.097	3.922	0.511	0.308
^{D}	0.096	3.906	0.557	0.035

Table 6: Business Cycle Statistics, United States, 1980Q1 - 2013Q4. Cyclical components are computed using HP lter with = 1600. denotes the labor wedge for the full turnover case (= 1). $^{S}_{n}$ and $^{D}_{n}$ are the \static" and \dynamic" labor market wedges. All wedges are computed assuming a Frisch elasticity of 0.18 and linear vacancy posting costs.

	y_{t-4}	y_{t-3}	y_{t-2}	y_{t-1}	y_t	y_{t+1}	y_{t+2}	y_{t+3}	y_{t+4}
					$= 0.66$				
s	0.134	0.111	0.078	0.033	-0.206	-0.097	-0.045	-0.057	0.039

$s = 0.66$ s s 0.134 0.134
S8]TJ/F8D10.9091 Tf 21.454 -11[(S8]- [(S8]TJ/F8are) -392 (the) -392 (9 (0static") -392 (and) -392 (9