CONSTITUTIONAL IMPLEMENTATION OF VERTICAL AND HORIZONTAL RESERVATIONS IN INDIA: A UNIFIED MECHANISM FOR CIVIL SERVICE ALLOCATION AND COLLEGE ADMISSIONS

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ABSTRACT. In order to address the historic discrimination faced by various communities under the caste system, a comprehensive affirmative action system exists in India, reserving access to government jobs and to enrollment in higher educational institutions. While there is a Supreme Court-mandated mechanism to implement these reservations when the positions are homogeneous, no mechanism is provided when the positions are heterogeneous.

1. Introduction

Affirmative action policies are implemented in India through a reservation system that earmarks up to 50 percent of positions at government jobs, and seats at publicly funded educational institutions, to the members of socially disadvantaged groups referred to as *reserved categories.* The three main reserved categories are:

(1) *Scheduled Castes (SCs)* whose members, being rated outside the caste system, faced centuries-long systematic discrimination,
(2) *Scheduled Tribes (STs)* whose members belong to indigenous ethnic groups of India, and
(3) *Other Backward Classes (OBCs)* whose members belong to castes which are educationally or socially disadvantaged.

The remaining members of the society are collectively referred to as the *general category.*[1] The reservations provided to the members of these three *reserved categories* are protected by the Article 16(4) of the Constitution of India, which reads:

> Nothing in this article shall prevent the State from making any provision for the reservation of appointments or posts in favor of any backward class of citizens which, in the opinion of the State, is not adequately represented in the services under the State.

While other forms of special provisions are also allowed under the Constitution, these reservations—referred to as *vertical.*

Given a merit ranking of candidates, it is straightforward to allocate the positions in a way it complies with *Indra Sawhney (1992)*, provided that:

(1) all positions are homogeneous, and
(2) vertical reservations are the only type of special provisions available.

First, the open positions are allocated to the candidates with the highest merit ranking from all categories, and next for each of the reserved categories; vertically reserved positions are allocated to the remaining candidates with the highest merit ranking from these categories. In most applications in India, however, the desired allocation procedure is less clear, because either

(1) the positions are heterogenous, or
(2) there are additional (but lesser) special provisions for other disadvantaged groups in the form of "minimum guarantees."

In a companion paper, Sönmez and Yenmez (2019), we present the challenges faced due to the second complication only, and offer a solution for this allocation problem when all positions are homogeneous. Naturally, the problem is more complex when positions are heterogeneous, and this is indeed reflected in the handling of numerous court cases where a large number of inconsistencies can be observed between the rulings, including those at the Supreme Court level. Focusing on this more demanding version of the problem, we present the additional (implementation and legal) challenges faced in India due to the heterogeneity of the positions, and propose a solution for the most general case that covers both the (possible) heterogeneity of the positions and all types of special provisions allowed in *Indra Sawhney (1992)*.

Although the principles that govern the implementation of reservation policies are articulated in great depth in the landmark judgement *Indra Sawhney (1992)*, a mechanism that implements this policy is not provided (either in this or any other Supreme Court judgement) when the positions are heterogeneous.[3] As a result, officials at various government institutions throughout India have been designing their own mechanisms, and often facing civil action either due to the failure to comply with the Supreme Court-mandated principles, or due to the confusion on the part of litigating parties. One widespread routine that contributes both to the confusion and to numerous lawsuits consists of,

(1) tentatively allocating open positions to candidates from all categories as a first phase of the allocation process, and

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[3]In contrast, a mechanism is not only provided by another historical Supreme Court judgement *Anil Kumar Gupta, Etc vs State Of Uttar Pradesh & Ors (1995)*, but also mandated throughout India, when all the positions are homogenous. The case is available at https://indiankanoon.org/doc/1055016/ (last accessed on 04/09/2019).

an open position to receive a more-preferred position reserved for his category, the next deserving candidate can be,

(1) a member of the general category who is either holding a less-preferred open position from phase 1, or remains unassigned, or

(2) another MRC candidate who is holding a less-preferred position from phase 1, or

(3) another member of a reserved category who remains unassigned at the end of phase 1.

Thus, the widespread practice of tentative allocation of the open positions in the first phase results in the creation of an artificial interim allocation, one that is often given too much weight despite being a technical construct. This in turn results in awarding the "property rights" of a vacated open position exclusively to the members of a specific category, creating an open invitation for a legal challenge. This misguided construction of property rights is the primary source of the dispute in a vast majority of legal conflicts involving MRC candidates. One of the lower court cases preceding the Supreme Court judgement in *Union of India vs Ramesh Ram & Ors (2010)* made a similar observation. The judges in the lower court case included the following statement in their ruling:[9]

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In doing so, the respondents also would notice that the steps taken by them
in accordance with the Rules 16 (3)(-)(5) are redundant once they issue
the result of recruitment in one phase, instead of two as they have become
primary cause for the litigation and avoidable confusion in the minds of the
candidates seeking recruitment.
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This judgement of the lower court also specified that, vertical reservations are to be respected for each job, and the principle of *inter se* merit has to be respected in the spirit of *Anurag Patel (2004)*. This judgement by the lower court, which is spot on, was not followed

can easily be designed through a simple adaptation of the celebrated *deferred acceptance algorithm* (Gale and Shapley 1962). In most practical applications in India, however, there are additional special provisions referred to as *horizontal reservations*, and designing a mechanism based on the deferred acceptance algorithm requires more care for this case. This more challenging version of the problem gained more prominence in the recent years, since many states in India adopted horizontal reservation for women in the last decade,[10] and a 3% horizontal reservation for the disabled is mandated by the Supreme Court judgement *Union Of India & Anr vs National Federation Of The Blind (2013)*.[11] Unlike vertical reservations that are implemented in the form of a "set aside," horizontal reservations are implemented in the form of a "minimum guarantee." Depending on the structure of horizontal reservations, complementarities between candidates may be introduced in the allocation problem, a condition that precludes a mechanism that is based on the deferred acceptance algorithm. We identify a necessary and sufficient condition in Section 5 on the structure of horizontal

lawsuits, but also contributed to inconsistent judgements in this context. We aim to fill this gap with the current paper.

Our paper is not the first one to suggest a mechanism based on the deferred accep-

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merit without using the benefits of vertical reservations, this position is not necessarily at his first choice job. Therefore, he would rather receive an SC-category position at a more-preferred job. At this point, the following important questions emerge, where the answers guide the mechanics of the rest of the mechanism:

(1) Shall an MRC candidate from class $X \in \{SC, ST, OBC\}$ be allowed to *migrate* to a higher choice job, receiving a category-X position?

(2) If the answer to the first question is in the positive, then what is to happen to the open-category position that was tentatively assigned to the MRC candidate?

These two questions and their answers are at the heart of countless lawsuits in India. We next present four Supreme Court cases in this context. Through these cases we argue that the concept of a meritorious reserved candidate is flawed, and it is the main source of the legal conflict and confusion in all of these cases and countless others. All these difficulties can be avoided with a more carefully designed mechanism that complies with *Indra Sawhney (1992)*, which we present in Section 4.[12]

2.2. Anurag Patel vs U.P. Public Service Commission (2004). The Uttar Pradesh Public Service Commission (UPPSC) conducted an examination in 1990, merit ranking all candidates, and used the following mechanism to allocate 358 positions at various jobs:

Step 1. Allocate the $\mathring{a}_{j45}$

Under the *UPPSC mechanism*, the open-category positions are allocated in the first step based on the merit ranking, and next in the second step the vertically reserved positions are allocated within each category to the remaining candidates of each reserved category.

At least one of the shortcomings of this mechanism is very apparent: MRC candidates who receive their assignments in Step 1 are not given an opportunity to migrate and be considered for any of the vertically reserved positions for their categories, and as such they often receive positions at less-preferred jobs compared to lower merit ranking candidates from their own categories. Therefore, the UPPSC mechanism fails to respect *inter se merit*, an important principle that plays a key role in all Supreme Court cases we discuss in Sections 2.2-2.4.[13] This shortcoming of the UPPSC mechanism resulted in a lawsuit at the High

Service Commission (UPSC), by conducting Civil Service Examinations periodically. Given the merit ranking produced by the Civil Service Examination along with the submitted preferences of the candidates over the set of jobs, the following *UPSC mechanism* is used to allocate the positions.

Step 1. Tentatively allocate the $\mathring{a}_{j \in J} r_j^O$ units of open-category positions to the candidates using the serial dictatorship induced by the given merit ranking.

Promote candidates from the reserved categories SC/ST/OBC who receive tentative positions at this step to the status of MRC.

Finalize all tentative assignments, except those received by the MRC candidates.

Step 2. For each of the classes $X \in \{SC, ST, OBC\}$, consider all category-X candidates (including the MRC candidates who received a tentative assignment in Step 1), and tentatively allocate the

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    iii) The amended Rule 16 (2) only seeks to recognize the inter se merit
between two classes of candidates i.e.  a) meritorious reserved category
candidates b) relatively lower ranked reserved category candidates, for the
purpose of allocation to the various Civil Services with due regard for the
preferences indicated by them.
    iv) The reserved category candidates ''belonging to OBC, SC/ ST
categories'' who are selected on merit and placed in the list of
General/Unreserved category candidates can choose to migrate to the
respective reserved category at the time of allocation of services.  Such
migration as envisaged by Rule 16 (2) is not inconsistent with Rule 16 (1)
or Articles 14, 16 (4) and 335 of the Constitution.
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Therefore, in the context of allocation of government jobs, the Supreme Court judgement *Ramesh Ram (2010)* provides the following answers to the questions posed in Section 2.1:

(1) An MRC candidate is entitled to migrate to a higher choice job claiming a position vertically reserved for his category.

(2) The open-category positions vacated by MRC candidates are to be offered to the general-category candidates.

The judges of the Supreme Court justified this important decision based on the principle of *inter se* merit, reaffirming the judgement in *Anurag Patel (2004)*. However, there is an important oversight in their judgement, one which makes the UPSC mechanism unconstitutional. While the Supreme Court overruled the judgement by the Madras High Court, justifying their decision based on the principle of *inter se* merit, the judges of the Supreme Court failed to observe that the UPSC mechanism itself does not comply with this important principle. The following simple example makes this point.

Example 1. There are three jobs x, y, z with a total of four positions. Each job has one open-category position, and job x has one additional OBC-category position. There are five candidates a_1, a_2, a_3, b_1, b_2, where candidates b_1, b_2 are members of OBC and candidates a_1, a_2, a_3 are members of the general category, who are not eligible for the OBC-category position. All candidates have identical preferences where x is their first choice, x

Candidate b_1 is the only MRC candidate. Assignment of candidate a_1 is finalized as an open position at job x, assignment of candidate a_2 is finalized as an open position at job z.

Step 2. OBC members b_1 and b_2 are the only candidates eligible for the one OBC-category position at job x. Candidate b_1, having higher merit ranking than candidate b_2, tentatively receives this position.

Step 3. One waitlist for the general category, and another for the OBC is prepared. There is only one MRC candidate, and, therefore, there is one candidate in each waitlist. Candidate a_3 is waitlisted at the general-category waitlist, and candidate b_2 is waitlisted at the OBC-category waitlist.

Step 4. MRC candidate b_1's assignment is finalized as the more-preferred job from Steps 1 and 2. He receives the OBC-category position at his first choice job x.

Step 5. The position vacated by the MRC candidate b_1 is an open-category position at job y. It is assigned to candidate a_3, the only candidate in the general-category waitlist.

Therefore, the final outcomes is

$$
\begin{pmatrix}
a_1 & a_2 & a_3 & b_1 & b_2 \\
x & z & y & x & \text{Æ}
\end{pmatrix}.
$$

Observe that this outcome does not respect *inter se* merit. Candidate a_2 receives a less-preferred assignment than candidate a_3, despite being a member of the same class (i.e. the general category) and having higher merit ranking.

Indeed, a close inspection of Example 1 reveals a number of additional issues with the judgement in *Ramesh Ram (2010)*. The Supreme Court ruled that:

> The seats vacated by MRC candidates in the General Pool will be offered to general-category candidates.

This is based on the assumption that, candidates from the general category will have a higher merit ranking than those in the reserved categories. As it is seen in Example 1, this may not always be the case. In our view, offering the vacated position to the lowest merit ranking candidate a_3 is not justified when the higher merit ranking candidate b_2 remains unassigned simply because he is a member of OBC. A system that is intended as positive discrimination for candidate b_2 results in discrimination against him. Equivalently, the *cut-off score*, the minimum score needed for a position, is higher in this example for the OBC candidates than for the general-category candidates. These types of scenarios result in some other related anomalies as well. In the absence of affirmative action, the outcome of the UPSC mechanism would have been

$$
\begin{pmatrix}
a_1 & a_2 & a_3 & b_1 & b_2 \\
x & y & \text{Æ} & x & z
\end{pmatrix},
$$

and the OBC candidate b_2 would have been better off. Or, alternatively, had candidate b_2 not declared his OBC membership, he would have again received a position at job z. This last point also shows that the UPSC mechanism is not *incentive compatible*.[16]

2.4. Tripurari Sharan & Anr. vs Ranjit Kumar Yadav (2018). The judgement in *Ramesh Ram (2010)*, discussed in Section 2.3, is now considered a main reference for allocation of government jobs when the positions are heterogeneous. Based on this reference judgement, the open-category seats vacated by MRC candidates are to be offered to the general-category candidates for allocation of government jobs. We emphasize "government jobs," because the Supreme Court has taken a completely opposite position for the allocation of seats at medical colleges. While the main reference for this application is considered to be *Shri Ritesh R. Sah vs Dr. Y.L. Yamul & Ors (1996)*, we instead discuss the more recent Supreme Court case *Tripurari Sharan (2018)*,[17] which is more illuminating for our purposes.

Citing the judgement in *Ramesh Ram (2010)*, the petitioners appealed in *Tripurari Sharan (2018)* an earlier decision by the Patna High Court, which ruled:

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In case of admission to medical institutions, an MRC can have in, for
the purpose of allotment of institutions, of his choice, the option of
taking admission in a college, where a seat in his category is reserved.
Though admitted against a reserved seat, for the purpose of computation
of percentage of reservation, he will be deemed to have admitted as an
open category candidate, rather he remains an MRC. He cannot be treated
to have occupied a seat reserved for the category of reservation he belongs
to.  Resultantly, this movement will not lead to ouster of the reserved
candidate at the bottom on the list of that reserved category.  While
his/her selection as reserved category candidate shall remain intact, he/she
will have to adjusted against remaining seats, because of movement of an MRC
against reserved seats, only(against)-525(reserved)-525(seats,)-525(only(Oiniin-250(Court,)
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seat only. There are five candidates a_1, a_2, b_1, b_2, b_3. Candidates b_1, b_2, b_3 are members of OBC, and candidates a_1, a_2 are members of the general category who are not eligible for the reserved positions. The candidates' preferences are given as

where the candidates submit their preferences, but it can also be used as a sequential mechanism where the candidates pick their choices one at a time following their merit rankings. Indeed, this feature of the serial dictatorship is utilized in some of the applications in India. The lawsuit brought to the Supreme Court in *Samta Aandolan Samiti (2013)*[18] is about one of these applications.

As in *Tripurari Sharan (2018)*, discussed in Section 2.4, the petition in *Samta Aandolan Samiti (2013)* also concerns the allocation of seats at medical colleges, and as such the precedent for this case is also *Shri Ritesh R. Sah (1996)*. The following sequential mechanism is

was one of the reasons this mechanism was challenged in *Samta Aandolan Samiti (2013)*. The following quote from the court proceedings illustrates the extent of this collusion:

> The petitioners aver that the respondents had conducted the counseling
> in strict adherence of the procedure quoted hereinabove. However, the
> respondents forced reserve candidates to obtain the unreserved (UR) seats
> by note (4.2.a) in counseling call letter. In this way the respondents
> deliberately tried to convert UR seats to reserve category seat because of
> note 4.2. Otherwise the candidates would have been provided freedom to opt
> seats under UR seats or category seats of their choice in different AIIMS.

In this way, members of OBC secured 45% of the seats even though they were reserved 27% of the seats. Ironically, the Supreme Court did not find any merit in the petition, dismissing the case.

3. The Case Against the MRC-Based Mechanisms

In Sections 2.3-2.5 we have argued that not only do the allocation mechanisms employed by various Indian institutions have important shortcomings, but also the Supreme Court judgements on these mechanisms have a number of inconsistencies. In this section, we argue that the source of all these difficulties is the flawed extension of the concept of meritorious reserved candidates from the homogeneous positions case to the heterogeneous positions case, and the artificial challenges this extension generates due to the concept of "migration," which is a byproduct of this flawed extension.

In the landmark Supreme Court judgement *Indra Sawhney (1992)*, reservations provided to historically discriminated classes of SC, ST, and OBC are explicitly specified in the form of a "set aside," in the sense that:

(1) unreserved positions are open for all, including for the members of SC, ST, and OBC, and

(2) an open position obtained by a member of any of these classes solely on the basis of merit is not counted against the positions reserved for these classes.

This type of reservation is very straightforward to implement when all the positions are identical. Open positions are allocated first on the basis of merit, to be followed by the reserved positions to reserve-eligible candidates (again on the basis of merit). Extending this idea to the case where the positions are heterogenous requires more care, and this is where a seemingly natural idea has not only resulted in the introduction of numerous poorly-behaved mechanisms in India, but it has also resulted in several inconsistent court decisions. Since the open positions are allocated prior to reserved positions when all positions are identical, one may be tempted to process them in this way as well when the positions are heterogeneous too. In doing so, higher merit ranking candidates from the

reserve categories are able to receive some of these open positions, albeit not necessarily at their first choices. This situation begs the questions posed in Section 2.1:

(1) Shall these candidates, known as meritorious reserved candidates, be allowed to *migrate* to higher choice jobs, claiming a position reserved for their respective classes?

(2) If they are allowed to migrate, then what happens to the open-category positions they received on the basis of their merit rankings?

While the first question was answered in the positive by the Supreme Court judgement in *Anurag Patel (2004)*, conflicting decisions were given for the second in the two Supreme Court judgements *Shri Ritesh R. Sah (1996)* and *Ramesh Ram (2010)*. However, observe that these questions are not about the fundamentals of the problem, but rather about the mechanics of a specific mechanism. Who is to say that open positions are to be processed before the reserved positions? Indeed, they can all be processed together, without creating any need for the concept of "migration" and the legal challenges it creates. Moreover, the term meritorious reserved candidate that applies uniformly for all the jobs is a misleading term, and in our view this terminology also contributes to the confusion. A candidate may be meritorious enough for one of the jobs but not necessarily for another.

It turns out that the confusion and the artificial problems caused by the tentative allocation of the open-category positions as a first step were observed by the judges of one of the lower courts, Central Administrative Tribunal, Chennai Bench (CAT-CB), prior to the Supreme Court judgement in *Ramesh Ram (2010)*. In particular, the judges have directed the Union of India to announce their outcome in one phase in a manner that respects *inter se* merit, without relying on the artificial concept of migration.[20] Even though the direction of the judges of the CAT-CB is the right approach to allocate the positions, it was never followed, possibly because it is not a straightforward problem to solve. Fortunately, we will be able to achieve this task in the next section, building on some standard methods in matching theory and market design.

4. Formal Model

In this section, we introduce our concepts and notation for the most comprehensive version of the problem.

4.1. Preliminary Definitions. There is a set of jobs J and a set of candidates A.[21] For every job $j \in J$, there are q_j identical positions. There are three reserve-eligible categories referred to as "Scheduled Castes" (SC), "Scheduled Tribes" (ST), and "Other Backward Classes" (OBC). These categories, referred to as reserved categories, is denoted by $R =$

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[20]See Appendix B for a comprehensive quote from this case.

[21]Depending on the application, J can also refer to the set of educational institutions, and A can refer to the set of students.

$\{SC, ST, OBC\}$. A candidate who does not belong to a reserved category belongs to the "General" category (G). The set of all categories is denoted by $\mathcal{C} = \{SC, ST, OBC,$

The third and final type of reservations is called *horizontal reservations*. These reservations are considered lower-level special provisions rather than vertical reservations, and by *Indra Sawhney (1992)* they are implemented on a minimum guarantee basis for candidates with certain traits. In their judgement of *Anil Kumar Gupta (1995)*

In order to describe the implementation of horizontal reservations, we first consider the case where there are no vertical reservations, or equivalently the case where all the positions are open. Recall that, horizontal reservations are implemented on a minimum guarantee basis, and if the allocation on the basis of merit ranking—the *meritorious outcome*—already satisfies the minimum guarantees, no adjustment is necessary. This important characteristic of horizontal reservations is clearly emphasized in *Indra Sawhney (1992)*, and any excessive adjustment from the meritorious outcome is deemed unconstitutional.

We next introduce a choice rule that deviates minimally from the meritorious outcome, when incorporating the horizontal reservations. To this end, we need some additional notation.

Let A A be a set of candidates who are applying to job j 11.9552 Tf 25.996 081t we ne.iious out-

Sönmez and Yenmez (2019) show that C_j^{hor} is the unique merit-maximal choice rule, which

$$m(c) \succsim_c m(c^{\ell}).$$

A matching respects *inter se* merit, if a candidate with a higher merit score never prefers the assignment of a lower merit score candidate with an identical category and set of traits to her own assignment. In other words, a candidate never loses a position to another candidate with lower merit score, provided that they are equally privileged. By *Anurag Patel (2004)*, any matching that fails to satisfy this requirement is unconstitutional in India.

Of course, losing a position to a higher-privilege candidate who has a lower merit score is even more objectionable than losing a position to an equally privileged candidate who has a lower merit score. This observation motivates the following property.

A matching m **eliminates justified envy** if, for any pair of candidates $c, c^{\ell} \in A$ with $r(c) \succeq r(c^{\ell})$, $t(c) \supseteq t(c^{\ell})$, and $s(c) > s(c^{\ell})$, we have

$$m(c) \succsim_c m(c^{\ell}).$$

Elimination of justified envy is a requirement even more vital than respecting *inter se* merit. It states that whenever (1) two candidates c and c^{ℓ} have the same category or c has a reserve-eligible category while c^{ℓ} is a general-category candidate, (2) c has any trait that c^{ℓ} has, and (3) c has a higher merit score than c^{ℓ}, c likes as a8/es the following property.

A matching m **eliminates**

Step k: Each candidate who was rejected in Step $k-1$ applies to her next preferred acceptable job, if such a job exists. Suppose that A_j^k is the union of the set of candidates who were tentatively accepted by job j in Step $k-1$, and the set of candidates who just proposed to job j. Job j tentatively accepts candidates in $C_j(A_j^k)$ and permanently rejects the rest. If there are no rejections, then stop.

DA produces a stable matching when choice rules satisfy the following two properties. The first is a basic rationality attribute: A choice rule C satisfies the **irrelevance of rejected candidates (IRC)** if for every set of candidates A and candidate $c \in A$, $c \notin C(A)$ implies $C(A \cap \{c\}) = C(A)$. In words, when a rejected candidate is removed from a set of applicants, the chosen set remains the same. See Aygün and Sönmez (2013) for a discussion of the irrelevance of rejected candidates.

The second property rules out complementarities between candidates.

Definition 2. *A choice rule C satisfies **substitutability** if for every set of candidates A and candidates $c, c' \in A$ with $c \neq c'$, $c \in C(A)$ implies $c \in C(A \cap \{c'\})$.*

Substitutability was introduced by Kelso and Crawford (1982) for matching markets with transfers. Substitutability, together with IRC, imply that DA produces a stable matching (Blair, 1988).[23]

A **mechanism** f takes a profile of candidate preferences as input and produces a matching. The outcome for candidate c at the reported preference profile $\succ_C = (\succ_c)_{c \in C}$ under mechanism f is denoted as $f_c(\succ_C)$. For any property on matchings, a mechanism satisfies the same property if, for every preference profile, the matching produced by the mechanism satisfies the property.

A mechanism f satisfies **strategy-proofness** if no candidate can misreport her preferences and get a strictly more-preferred job. More formally, for every candidate c and preference profile $\succ_C$, there exists no preference $\succ_c'$ such that $f_c(\succ_c', \succ_{C \cap \{c\}}) \succ_c f_c(\succ_C)$.

Even when choice rules satisfy substitutability and IRC, DA does not have to be strategy-proof. To satisfy strategy-proofness, the following property is needed: A choice rule satisfies **the law of aggregate demand** (LAD) if the number of candidates chosen from a set is weakly greater than that of any of its subsets. Mathematically, a choice rule C_j satisfies LAD if, for every $A \subseteq A' \subseteq A$, $C_j(A) \leq C_j(A')$.[24] DA is stable and strategy-proof when choice rules satisfy substitutability and LAD (Hatfield and Milgrom, 2005).

A candidate

For example, an OBC female candidate can withhold some of her reserve-eligible privi

FIGURE 1. The tree of traits in Example 3

node representing divorced. However, if there is a female candidate with disability, then she has to choose whether she would like to use the benefits of horizontal reservations for disability or for woman. Otherwise, the traits would not be nested, a situation that introduces complementarities in the problem and the possible non-existence of a stable matching.

Nested traits are common in India, and indeed it is Supreme Court-mandated for the case of the trait "disability." A typical requirement in India is, "at least 3% of the positions are reserved for the disabled, of which at least 1% each for vision disabled, hearing disabled, and locomotor disabled." Due to the rounding of the horizontally reserved positions, the total reservation for disabled often exceeds the sum of reservations for vision disabled, hearing disabled, and locomotor disabled, making the disability a non-trivial application of the nested structure.

When traits are nested, we assume that, for every trait t, the number of positions reserved for trait-t

If there are no remaining candidates or positions, stop and return the chosen set of candidates.

Step k (k ≥ 2): If there are no remaining traits, choose candidates with the highest merit scores for the empty positions. Otherwise, for every trait t that is not an ancestor of another trait, if there are less than r_j^t trait-t candidates in the set of applicants A, choose all of them. Otherwise, choose r_j^t trait-t candidates with the highest merit scores. Reduce the number of positions and the number of horizontal reservations for any ancestor trait of t by the number of chosen trait-t candidates. Remove t from the set of traits. If there are no remaining candidates or positions, stop and return the chosen set of candidates.

We are ready to present our first result.

Theorem 1. *Suppose that traits are nested. Then C_j^{hor} is equivalent to C_j^{nest}.*

This characterization provides an efficient way of constructing C_j^{hor} when traits are nested. First, consider the set of traits at the bottom of the tree of traits (i.e., those traits that are not an ancestor of any other trait). For each one of these traits, choose the candidates with these traits up to the number of reservations for this trait. Then update the number of remaining positions and reservations for the rest of the traits and repeat this procedure.[27]

Next, we present a result that clarifies the central role of nested traits in our design.

Theorem 2. *C_j^{hor} satisfies substitutability for every job allocation problem that has a candidate with no trait if, and only if, traits are nested.*

One implication of this result is the following.

Corollary 1. *$C_j^{v\&h}$ satisfies substitutability for every job allocation problem that has a candidate with no trait if, and only if, traits are nested.*

Together with IRC, this result implies that DA produces a stable matching when, for each job j, $C_j^{v\&h}$

7. Consequences of the Constitution (103rd Amendment) Act, 2019

In a highly debated reform on the reservation system, the *One Hundred and Third Amendment of the Constitution of India* provides ten percent reservation to the economically weaker sections (EWS) in the general category.[28] While the language of the act is not clear about whether the EWS reservation is vertical or horizontal, a government memorandum dated 01/31/2019 specifies it as a vertical reservation:[29]

> 7.   ADJUSTMENT AGAINST UNRESERVED VACANCIES:
>
>   A person belonging to EWS cannot be denied the right to compete for appointment against an unreserved vacancy. Persons belonging to EWS who are selected on the basis of merit and not on account of reservation are not to be counted towards the quota meant for reservation.

If the One Hundred and Third Amendment survives the Supreme Court challenge and, implemented as a vertical reservation, it will likely amplify the legal challenges faced due to MRC-based mechanisms.

It is estimated that, around 26% of the population in India does not belong to the Other Backward Classes (OBC), Scheduled Castes (SC) and Scheduled Tribes (ST) categories.[30] Therefore, in the absence of the new amendment, about 26% of the population belongs to the general category. While the amendment is intended for the economically weaker sections of the general category, according to most estimates more than 80% of the members of this group satisfy the eligibility criteria for the EWS reservation.[31] This means, with the introduction of the EWS reservation, the fraction of the population who are ineligible for any vertical reservation reduces to roughly 5-6% of the population. Therefore, the "new

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[28]The bill of the *One Hundred and Third Amendment of the Constitution of India* was introduced in the Lok Sabha—the lower house of the Parliament of India—on 01/08/2019 as the Constitution (One Hundred and

general category," those members of the society who are ineligible for any vertical reservations, shrinks to approximately 5-6% of the whole population.[32] This observation, by

the general merit list but is getting a cadre of her choice as a reserved
candidate, she is counted as reserved), say bureaucrats. So, many who are

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Case 4: Consider the case when $c^\ell \in C^{nest}(A)$ and c^ℓ is chosen at a node for trait t^ℓ such that t^ℓ is not an ancestor of t and t is not an ancestor of t^ℓ. Then c is still chosen when t is considered at Step k from $A \cap \{c^\ell\}$, so $c^\ell \in C^{nest}(A \cap \{c^\ell\})$.

Next we show necessity. Suppose that traits are not nested. Since traits are not nested, there exist distinct traits $t, t^\ell \in T$ and candidates $c_1, c_2, c_3 \in A$ such that $\{t, t^\ell\} \subseteq t(c_1)$; $t \in t(c_2), t^\ell \notin t(c_2)$; and $t^\ell \in t(c_3), t \notin t(c_3)$. In addition, there exists a candidate $c_4 \in A$ with no traits, i.e., $t(c_4) = \emptyset$.

Let $q_j = 2$, $r_j^t = r_j^{t^\ell} = 1$, and $s(c_4) > s(c_3) > s(c_2) > s(c_1)$. Then $c_1 \in \{c_1, c_4\} = C_j^{hor}(\{c_1, c_2, c_3, c_4\})$ and $c_1 \notin \{c_2, c_3\} = C_j^{hor}(\{c_1, c_2, c_3\})$. Therefore, C_{jj}^{hor}

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Appendix B. A Quote from Ramesh Ram (2010)

The following quote is given in *Ramesh Ram (2010)*:

Central Administrative Tribunal, Chennai Bench in O.A. No. 690 of 2006 and 775 of 2006 had given the following directions:

''(i) The impugned Rule 16 (2) is declared as valid so long as it is confined to allocation of services and confirms to the ratio of Paras 4 to 6 of Anurag Patel order of the Hon'ble Apex Court.

(ii) The Supplementary List issued by the second respondent to the first respondent dated 3.4.2007 is set aside. This would entail issue of a fresh supplementary result from the reserved list of 64 in such a way that adequate number of OBCs are announced in lieu of the OBCs who have come on merit and brought under General Category. The respondents are directed to rework the result in such a way the select list for all the 457 candidates are announced in one lot providing for 242-general, 117 OBC, 57 SC and 41 ST and also ensure that the candidates in OBC, SC & ST who come on merit and without availing any reservation are treated as general candidates and ensure that on equal number of such reserved candidates who are of merit under General Category, are recruited for OBC, SC & ST respectively and complete the select list for 457. Having done this exercise, the respondents should apply Rule 16 (2) to ensure that allocation of the service is in accordance with rank-cum- preference with priority given to meritorious reserved candidates for service allocation by virtue of Rule 16 (2) which is as per para 5 of Anurag Patel order. The entire exercise, as directed above, should be completed as per the order.

(iii) Applying the ratio of Anurag Patel decision of Hon'ble Apex Court (Paras 6 & 7), if there is need for re-allocation of services, the respondents will take appropriate measures to that extent and complete this process also within two months from the date of receipt of a copy of this order."

The CAT had also issued the following direction as to how the results of the UPSC examinations (2005) should have been announced:

''If the UPSC had followed the decision of the Hon'ble Apex Court cited supra and released the select list in one go for all the 457 vacancies then it would have ensured that the select list contained not only 117 OBCs but also an additional number of OBC candidates by this number, in additional to 117 under 27% reservation, while simultaneously be number of general candidates recruited will be less to the extent of OBCs recruited on merit and included in the general list in the result of Civil Services

Examination, 2005.   Once this order is met, the successful candidates list will include 242 candidates in the General Category which is inclusive of all those Reserved Category candidates coming on merit plus 117 OBC, 57 SC and 41 ST exclusively from these respective reserved categories by applying relaxed norms for them..   If such a list is subjected to Rule 16(2) of Civil Services Examination, 2005 in present form for making service allocation only and then services are allotted based on Rule 16(2) in this context, then the announcement of recruitment result and allocation services will be both in accordance with law as per various judgments the Hon'ble Apex Court and in accordance with the extent orders issued by the Respondent No.1 and also in keeping with spirit of Rule 16 (2) so that, the meritorious reserved candidates get higher preference service as compared to their lower ranked counter parts in OBC, ST,SC. In doing so, the respondents also would notice that the steps taken by them in accordance with the Rules 16 (3)(-)(5) are redundant once they issue the result of recruitment in one phase, instead of two as they have become primary cause for the litigation and avoidable confusion in the minds of the candidates seeking recruitment.''