Two-Sided Matching via Balanced Exchange

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1 Introduction

We introduce and model a new class of two-sided matching markets without explicit transfers, in which there is an additional fundamental constraint.[1] The eventual market outcome is linked to an initial status-quo matching, which may give participants certain rights that constrain how future activity can play out. Since market outcome is typically different from the status quo, such activities loosely resemble an exchange in which one side of the agents are changing or acquiring new partners in addition to the two-sided matching market structure. In such markets, a fundamental balancedness condition needs to be sustained with respect to the status-quo matching. The motivation for such a balancedness constraint can be different depending on the features of the market. Two contrasting examples are labor and higher education markets, where workers and colleges provide services to be compensated, respectively. In worker exchange, a worker needs to be replaced with a new one at her home firm so that this firm can function properly, and thus, the market needs to clear in a balanced manner. In student exchange, the college that is matched with an exchange student should be able to send out a student as well so that its education costs do not increase, and thus, the market needs to clear in a balanced manner. There are several prominent examples of such exchanges, such as national and international teacher-exchange programs, clinical-exchange programs for medical doctors, worker-exchange programs within or across firms, and student-exchange programs among colleges. This balancedness constraint induces preferences for firms/colleges not only over whom they get matched with (i.e., import), but also over whom they send out (i.e., export). The most basic kind of such preferences requires the firm/college to have a preference for balanced matchings, i.e., for import and export numbers to be equal. We analyze our model over two explicit market applications: (permanent) tuition exchange and temporary worker exchange (see Section 2 for details).

In tuition exchange

their preferences over matchings are determined through their rankings over the incoming class and how balanced the eventual matching is.[2] We start by showing, through a simple example, that individual rationality and nonwastefulness, standard concepts in two-sided matching markets, and balancedness are in general conflicting requirements (Proposition 1). For this reason, we restrict our attention to the set of balanced-efficient mechanisms. Unfortunately, there exists no balanced-efficient and individually rational mechanism that is immune to preference manipulation for colleges (Theorem 2).

Although 2S-TTC is balanced-efficient, it may not match the maximum possible number of students while maintaining balance. We show that if the maximal-balanced solution is different from the 2S-TTC outcome for some preference profile, it can be manipulated

Compared to deferred-acceptance-based current practice, they show that a TTC-based approach doubles the number of teachers moving from their initial assignment. Additionally, when the distribution of the ranks of teachers over the schools are considered, the outcome of the TTC-based approach stochastically dominates that of the current practice. Thus, there exist real-life settings, in which our proposals can lead to significant welfare improvements.

We extend this model for temporary worker exchanges, such as teacher-exchange programs. We tweak our model slightly and assume that the quotas of the firms are fixed at the number of their current employees, and, hence, firms would like to replace each agent who leaves. We also assume that firm preferences are coarser than colleges in tuition exchange due to the temporary nature of the exchanges. We assume they have weakly size-monotonic preferences over workers: larger groups of acceptable workers are weakly better than weakly smaller groups of acceptable workers when the balance of the matching with larger groups of acceptable workers is zero and the balance of the matching with smaller group of worker is nonpositive.[9]

Many colleges give qualified dependents of faculty tuition waivers. Through a tuition-exchange program, they can use these waivers at other colleges and attend these colleges for free. The dependent must be admitted to the other college

rent form.[18] The Jesuit universities exchange program FACHEX is another one that is adversely a□ected. The program still does not have an explicitly embedded balancedness requirement. It includes all Jesuit universities but Georg students by c. Let $□_C = (□_c)_{c□C}$ be the list of college **internal priority orders**, where $□_c$ is a linear order over S_c

In a revelation game, students and colleges report their preferences; additionally, colleges report their admission and eligibility quotas.[26] A mechanism ϕ is **immune to preference manipulation for students** or **colleges** if for all $[q, e, \succsim]$, there exists no $i \in S$ (or $i \in C$) and $\succsim'_i$ such that $\phi[q, e, (\succsim'_i, \succsim_{-i})](i) \succ_i \phi[q, e, \succsim](i)$. A mechanism ϕ is **immune to preference manipulation** if it is immune to preference manipulation for both students and colleges. A mechanism ϕ is **immune to quota manipulation** if for all $[q, e, \succsim]$, there exists no $c \in C$ and (q'_c, e'_c) with $q'_c \leq q_c$ such that $\phi[(q'_c, q_{-c}), (e'_c, e_{-c}), \succsim](c) \succ_c \phi[q, e, \succsim](c)$. A mechanism ϕ is **strategy proof for colleges** if for all $[q, e, \succsim]$, there exists no $c \in C$ and $(q'_c, e'_c, \succsim'_c)$ with $q'_c \leq q_c$ such that $\phi[(q'_c, q_{-c}), (e'_c, e_{-c}), (\succsim'_c, \succsim_{-c})](c) \succ_c \phi[q, e, \succsim](c)$. A mechanism is **strategy proof for students** if it is immune to preference manipulation for students. A mechanism is **strategy proof** if it is strategy-proof for both colleges and students.[27] A mechanism ϕ is **group strategy proof for students** if for all $[q, e, \succsim]$, there exists no $S' \subseteq S$ and

Throughout our analysis, we impose a weak restriction on college preferences. Assumption 1 below states that a college prefers a better scholarship class with zero net balance to an inferior scholarship class with a nonpositive net balance.

Assumption For any μ, ν □ M^u and c □ C, if $b_c^\mu = 0$, $b_c^\nu \le 0$, and $\mu(c) P^c$

2017). In contrast, in our market, college slots are not objects. Therefore our definition of a mechanism, and the properties of matchings and mechanisms (except strategy-proofness for students) do not have any analogous translation in such problems. However, because

Theorem Under Assumption 1 and when true eligibility quotas satisfy $e_c = |S_c|$ for all $c \in C$, 2S-TTC is immune to quota manipulation.

We prove the theorem with a lemma showing that as the quotas of a college increase, the import and export sets and the admitted class of students of this college also (weakly) expand under 2S-TTC.[32]

Theorems 3 and 4 point out that only colleges can benefit from manipulation, and they can manipulate by misreporting their preferences. Moreover, the only way to manipulate preferences is to report an acceptable student as unacceptable. Suppose we take all the admitted students in the regular admission procedure as acceptable for a tuition-exchange scholarship. Then, to manipulate 2S-TTC, a college needs to reject a student who satisfies the college admission requirements. Usually college admission decisions are made before the applicants are considered for scholarships.[33]

Proposition 2 below implies that colleges do not benefit from misreporting their ranking over incoming classes.

Proposition Under Assumption 1, colleges are indifferent among strategies that report preferences in which the same set of students is acceptable with the same quota report under the 2S-TTC mechanism.

We have shown that 2S-TTC has appealing properties. In the following theorem, we show that it is the unique mechanism satisfying a subset of these properties.

Theorem 5 Under Assumption 1, 2S-TTC is the unique student-strategy-proof, acceptable, and balanced-efficient mechanism that also respects internal priorities.

In the proof of our characterization theorem, we use a different technique from what is usually employed in elegant single quota characterization proofs such as Svensson (1999) and Sönmez (1995) for the result of Ma (1994). Our proof relies on building a contradiction with the claim that another mechanism with the four properties in the theorem's

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[32] or s n star ontrast w s ar r su ts n t ratur or stab an s s stu nt an o opt a stab an s s ar pron to a ss on quota an pu at on by o s v n un r r spons v pr r n s s n an Kon an nv r us C pr s nts a robust r y or a o on prob s n n ntra a ss ons at us stu nt opt a stab an s an a so n tu t on x an n a ntra ar t s or s an n n App n x A

[33] ur proposa a so pr v nts so o r an pu at on poss b t s For xa p r t now a o r a y s on o ts own stu nts n t ay r as ts xport quota pr v nt n s stu nt to b b an stu nt n n att n s to r o o rou tu t on r ss on How v r n our proposa a o 's xport quota a so t r n s s t o ts own stu nts w o ar b or

hypothesis can exist. Suppose such a mechanism exists and finds a different matching than 2S-TTC for some market. The 2S-TTC algorithm runs in rounds in which trading cycles are constructed and removed. Suppose $S(k)$ is the set of students removed in Round k, while running 2S-TTC in such a way that in each round only one arbitrarily chosen cycle is removed and all other cycles are kept intact. We find a Round k and construct an auxiliary market with the following three properties: (1) Eligibility quotas of home colleges of students in $S(k)$ are set such that these are the last certified students in their respective home institutions; (2) all preferences are kept intact except those of students in $S(k)$, whose preferences are truncated after their 2S-TTC assignments; and (3) all students in $S(k)$ are assigned $c_\emptyset$ under the alternative mechanism, while all students removed in the 2S-TTC algorithm before Round k have the same assignment under 2S-TTC and the alternative mechanism. This contradicts the balanced-efficiency of the alternative mechanism: we could give the students in $S(k)$ their 2S-TTC assignments while keeping all other assignments intact and obtain a Pareto-dominating balanced matching. Round k and the auxiliary market are constructed in three iterative steps.

Among all the axioms, only the respect for internal priorities is based on exogenous rules. One might suspect that more students will benefit from the tuition-exchange program if we allow the violation of respect for internal priorities. However, such mechanisms turn out to be manipulable by students.

Theorem 6 Any balanced and individually rational mechanism that does not assign fewer students than 2S-TTC and selects a matching in which more students are assigned whenever such a balanced and individually rational outcome exists, is not strategy-proof for students, even under Assumption 1.

4.1 Market Implementation: Tuition Remission and Exchange

Incorporating tuition-remission l29552(s(t07Tc1nc)50(e)50(d)c)2uT u50(d)bp50(d)piooo in a semi-decentralized fashion: first, colleges announce their tuition-exchange scholarship quotas and which of their students are eligible to be sponsored for both exchange and remission; then, eligible students apply for scholarship to the colleges they find acceptable; then colleges send out scholarship admission letters. At this stage, as students have also learned their opportunities in the parallel-running regular college admissions market, they can form better opinions about the relative ranking of the null college, i.e., their options outside the tuition-exchange market. Students submit rankings over the colleges that admitted them with a tuition-exchange scholarship and the relative ranking of their outside option. Finally, 2S-TTC is run centrally to determine the final allocation.

4.2 Allowing Tolerable Imbalances

Some programs care about approximate balance over a moving time window. Here, we relax the zero-balance constraint and allow each $c \in C$ to maintain a balance within an interval $[\ell_c, u_c]$ where $\ell_c \leq 0 \leq u_c$.[34] When either ℓ_c or u_c equals zero for all $c \in C$, the market turns into the case studied in Section 4. Let $(\ell_c, u_c)_{c \in C}$ be the tolerance profile.

When the colleges hold a non-zero balance, then there may exist some colleges exporting (importing) more than they import (export). Then, we cannot represent all allocations by cycles. Therefore we need to consider chains in addition to the cycles. A which consider her acceptable, and c

Theorems 7 and 8 hold without any assumptions on preferences. Under a mild assumption on college preferences, we can show that 2S-TTTC is individually rational and it induces a dominant-strategy equilibrium for colleges' quota reporting game to certify all their students and report their true admission quota.

Although 2S-TTTC is defined in a static problem, we can easily extend it to the dynamic environment where the aggregate balance over years matters. In particular, for each period t and $c \in C$ we can set counter b_c equal to c's aggregate balance in period $t-1$ where the aggregate balance in period $t-1$, is equal to the sum of balances between period 1 and $t-1$. Moreover, the exogenous priority rule used in period t can be determined based on the aggregate balance colleges carry at the end of period $t-1$ such that the highest priority can be given to the college with the highest aggregate balance and so on.

5 Temporary Worker Exchanges

Many organizations have temporary worker-exchange programs that can be modeled through our balanced two-sided matching framework. The first difference between such programs and tuition exchange is that these exchanges are usually temporary. Each firm usually requires a set of specific skills, e.g., a mathematics teacher to replace their own mathematics teacher. Compatibility and ability to perform the task are the main preference criterion rather than a strict preference ranking. E.g., finding a good teacher with a specific degree is the first-order requirement, rather than finer details about the rankings of all good teachers.

The second difference is that each position and each worker should be matched, unlike the tuition-exchange application. The workers are currently working for their home firms. Thus, the firms consider these workers necessarily acceptable. By contrast, in tuition exchange, colleges are not required to admit all the dependents of their employees. In temporary worker exchanges, a worker who does not want to go to a different firm necessarily stays employed in her home firm. We need to use a variant of the tuition-exchange model to facilitate balanced-efficient trade in such circumstances.

We can use the model introduced in Section 3 with slight changes. Since each firm accommodates its current workers, $q_c = |S_c|$ for each $c \in C$fi g T (c) 9 1 9 9 5 5 # e (c) 9 1 9 9

Our paper, besides introducing a new applied problem and proposing a solution to it, has six main theoretical and conceptual contributions: We introduce a new two-sided matching model that builds on the two most commonly used matching models in the literature: discrete object allocation, including school

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Supplementary Material for

“Two-Sided Matching via Balanced Exchange”

by

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Appendix A On Current Practice of Tuition Exchange

In t s append x, we ana yze t e current pract ce of tu t on exc ange. As t e centra zed process s oose y contro ed, once eac co ege sets ts e g b ty ad ss on quota and e g b e students are deter ned, t e ar et funct ons or su pt on states t at a better ad tted c ass s preferab e as ong as t e net ba ance does not decrease, ad ss on of unacceptab e students deter orates t e ran ngs of uncon stra ned atc ngs regard ess of t e r net ba ances, and a co ege dee s ts own students unacceptab e n tu t on exc ange Assu pt on ntroduces negat ve net ba ance averse preferences In a resu ts n t s sect on we w use Assu e prove t s propos t on by construct ng an assoc ated Ga e S ap ey co ege ad ss ons ar et n w c t e set of Ga e S ap ey stab e atc ngs s

eore s and do not conduct an equ br u ana ys s n a quota deter nat on ga e But t ey do po nt out t at n a fr ct on ess ar et t e co eges t at w be e y to ave a negat ve ba ance w be conservat ve and w decrease t e r e g b ty quotas for exports w c w furt er deter orate t e ba ances of ot er co eges

yp ca y no co ege fu y w t draws n pract ce as t ere s often a n u quota of part c pat on n p ace e con ecture t at t s cou d be nst tuted because of t e reasons out ned above G ven t at cont nued e bers p s an attract ve bene t often t es s a er co eges w announce t at t ey w port and export at t s n u quota requ re ent and w cont nue to be a e ber of t e progra w t out fu y w t draw ng fro t e syste

e conc ude t at under a new des gn for tu t on exc ange t ere s ou d be no roo

Acceptability: Students w be ass gned to nu co ege $c_\emptyset$ w enever t ey po nt to t, and ence, t ey w never need to po nt to an unacceptab e co ege. Hence, a student cannot be ass gned to an unacceptab e co ege. Moreover, a student cannot po nt to a co ege t at cons ders er unacceptab e. erefore, t e students ran ed be ow $\emptyset$ n P_c cannot be ass gned to c. us, S C s acceptab e.

Individual Rationality: Since eac s s ass gned to an opt on wea y better t an $c_\emptyset$, s does not nd v dua y b oc . Once a students n (c) are ran ed above $\emptyset$ n P_c for eac c

students w o are n t e cyc es re oved n ound $\leq K$ of C w ere K s t e ast round of C[10] e w prove t at s ba anced e c ent n two parts

Part I: e rst prove t at cannot be Pareto do nated by anot er acceptab e ba anced atc ng If s . ( ) t en (s) C $c_\emptyset$ s t e g est ran ed opt on n P_s t at cons ders er acceptab e at s no student s . ( ) can be ass gned to a better co ege cons der ng er acceptab e If t ere ex sts a atc ng suc t at $_s$ t en (s) cons ders s unacceptab e at s cannot be Pareto do nated by anot er acceptab e atc ng n w c at east one student n . ( ) s better o n

If a student s . ( ) s not ass gned to a ore preferred c C t at cons ders er acceptab e t en c s ou d be re oved n ound Let be an acceptab e and ba anced atc ng suc t at $(s) = c$ Suppose t ere ex sts anot er student s' suc t at $(s') = c$ and (s') c Note t at s' s an e g b e student Because s' s ass gned n ound $(s') = c$ s er favor te co ege a ong t e ones cons der ng er acceptab e at s n any acceptab e and ba anced atc ng n w c s s ass gned to (s') s' w be ade worse o Suppose $(s') = c$ for any s' (c)[11] en c

us, ∮ D C s group strategy proof for students ∎

e fo ow ng e a s used n prov ng eore

Lemma 1

Proof of Theorem 4. e prove a stronger vers on of eore . nder C suppose t at preference pro es are xed for co eges suc t at no co ege reports an un acceptab e student as acceptab e n ts preference report In t e nduced quota report ng ga e under Assu pt on t s a do nant strategy equ br u for a $c \in C$ to cert fy en any acceptab e ec an s w ass gn er to $c_\emptyset$ If |. (1)| 1 or |. (1)| = 1 and t e student n . (1)

one cyc e

Step 1: Construct a preference pro e $\tilde{\succsim}$ w t assoc ated ran ng $\tilde{P}$ as fo ows Let student s □ . c ran on y (s) as acceptab e n $\tilde{P}_s$ and $\tilde{\succsim}_j = \succsim_j$ for a □ $[(C$ □ .) \ $s]$ By t e execut on of t e C a gor t , C w se ect for $[q\ e\ \tilde{\succsim}]$ nce s strategy proof for students and acceptab e $[q\ e\ \tilde{\succsim}](s) = c_\emptyset$

en we c ec w et er t e ass gn ents of students n $\square^{k-}_{k'=}$. (′) are t e sa e n $[q\ e\ \tilde{\succsim}]$ and . If not, t en for so e ˜ , t ere ex sts a student $\tilde{s}$ □ . (˜) prefer r ng ($\tilde{s}$) to $[q\ e\ \tilde{\succsim}](\tilde{s})$ and eac student n $\square^{k-}_{k'=}$. (′) gets t e sa e co ege n and $[q\ e\ \tilde{\succsim}]$ en we repeat Step by ta ng $\succsim := \tilde{\succsim}$ $s := \tilde{s}$ and $:= \tilde{}$.

s repet t on w end by t e n teness of rounds en a students n $\square^{k-}_{k'=}$. (′) get t e sa e co ege n , e C outco e n $[q\ e\ \tilde{\succsim}]$ and $[q\ e\ \tilde{\succsim}]$, t en we proceed to Step

Step 2: In Step we ave s own t at s prefers (s) to $[q\ e\ \tilde{\succsim}](s) = c_\emptyset$ Suppose c s t e o e co ege of s et a new e g b ty quota $\tilde{e}_c$ equa to t e ran of student s n c s nterna pr or ty order, t at s, $\tilde{e}_c = r_c(s)$, and et

co eges, w c cons der er acceptab e and $c_\emptyset$ If a student s ass gned n t s round, t en s e s ou d get t e sa e co ege n Now cons der students ass gned n ound ′ w en 1 A t e co eges t at a student prefers to er ass gn ent and cons der er acceptab e s ou d ave been re oved or beco e non port ng n an ear er round e cannot a e t s student better o by ass gn ng er to a co ege t at cons ders er

Once we can run S C n t a y ass gn ng a ne g b e wor ers to t e r o e r s t e proof of eore p es t e wor er strategy proofness of S C

e proof of eore for un queness o ds w t a s g t c ange F rst note t at any Pareto e c ent student strategy proof and acceptab e ec an s ass gns wor ers to e t er t e r o e r s or better r s t at cons der t e acceptab e In t e un queness part of t e proof e eore s proof adopted for S C be ng t e on y ec an s sat sfy ng Pareto e c ency student strategy proofness acceptab ty and respect for n terna pr or t es n t e te porary wor er exc ange ode w e updat ng wor er s s preferences n Step we do t as fo ows ran (s) and er o e r as on y acceptab e r s n t e correct order of er true preferences And t en at t e end of Step s e w be ass gned to er o e r under Snce respects nterna pr or t es and s acceptab e student strategy proof and ba anced e c ent s w re a n at er o e r n Step en we reac Step we w ave a set of wor ers w o are ass gned to t e r o e r s by owever a trad ng cyc e between t e wou d prove tota we fare w t out v o at ng ba ancedness or feas b ty

Immunity to Preference Manipulation by Colleges: eca t at n any atc ng ba ancedness s sat s ed and r s t e r ad ss on quotas Hence under Assu pt on r s are nd erent between any acceptab e atc ng Snce t e S C ec an s se ects an acceptab e atc ng w en r s report trut fu y r s cannot be better o by an pu at ng t e r preferences over t e atc ngs and report ng quotas d erent fro t e r true quotas

Stability: Cons der an arb trary ar et $[q\ e\ \succsim]$ Denote t e outco e of S C by eca t at $q_c = |\ _c|$ for a $c \in C$ a wor ers cons der t e r current r s acceptab e a r s cons der t e r current wor ers acceptab e and wor ers w o are not cert ed re a n at t e r current r s Hence $|\ (c)| = q_c$ for a $c \in C$ Snce n a r s quotas are ed s nonwastefu Note t at any utua dev at on of wor er r pa r needs to end up w t a ba anced atc ng Snce a e p oyees n (c) are acceptab e rep ac ng one of t e e p oyees n (c) w t anot er one n $\setminus\ (c)$ cannot a e c better o Hence cannot be b oc ed by a wor er r pa r ■

Appendix C Tuition-Exchange Programs

e rst exp a n w y tu t on exc ange progra s ex st n t e rst p ace because so e co eges c oose to subs d ze facu ty d rect y nstead of part c pat ng n tu t on exc ange progra s A t oug t s ay create ex b ty for t e students any d rect co pensat on over s taxab e nco e w ereas a tu t on exc ange sc o ars p s not [23] u t on exc ange s not cons dered to be an nco e transfer [24] Moreover co eges ay not want to sw tc to suc d rect co pensat on progra s fro a cost sav ng perspect ve regard ess of t e tax bene t to t e facu ty e ber e present a s p e bac of t e enve ope ca cu at on to de onstrate t ese cost sav ngs ere are ore t an year co eges n t e and at ost a f of t e ave e bers p to at east one tu t on exc ange progra Suppose n students are g ven tu t on exc ange re ss on sc o ars ps a year Instead f a co ege nances t e tu t on of a facu ty e ber s c d t roug d rect cas co pensat on t en a tu t on exc ange co eges w ave to pay $\$n\bar{}$ w ere $\bar{}$ s t e average fu tu t on cost of co eges However assu ng t at average qua t es and s zes of co eges w t and w t out tu t on sc o ars p are t e sa e on y a f of t ese students w attend a tu t on exc ange co ege n return so t e co eges w on y get bac $\$\frac{n\bar{T}}{2}$ e re a n ng $\frac{n}{2}$ s ots w be ed w t regu ar students egu ar students on average pay about a f of t e tu t on t an s to ot er nanc a a d progra s For exa p e u t on D scount ng Study of t e Nat ona Assoc at on of Co ege and n vers ty Bus ness O cers report t at nco ng fres en pay on average 56% of fu tu t on at a pr vate un vers ty us t ey w on y pay $\$\frac{n\bar{T}}{}$ to tu t on exc ange co eges As tu t on ex c ange sc o ars ps const tute a very s a port on of co ege ad ss ons t s ca cu at on assu es t at average tu t on pay ent wou d not c ange by estab s ent of d rect cas co pensat on nstead of tu t on exc ange us as a resu t t e co eges w ose n tota about $\$\frac{n\bar{T}}{}$ w c corresponds to one fourt of average fu tu t on per student us t e tota per student sav ngs for t e facu ty e ber and t e co ege s ore t an a f of tu t on pay ent assu ng one t rd of t e d rect co pensat on s pa d n nco e tax at t e arg n by t e parent

The Tuition Exchange Inc (TTEI):3(a92(3(6(m)3(e)4(n)3g)120011.955297.1991440.3999 year en eac facu ty e ber sub ts t e EI app cat on to t e reg strat on o ce of t e r co ege If t e nu ber of app cants s greater t an t e nu ber of students t at t e co ege s w ng to cert fy t en t e co ege dec des w o to cert fy based on years of serv ce or so e ot er cr ter on nterna pr or ty order

Eac student w o s cert ed e g b e sub ts a st of co eges to t e a son o ce of er o e nst tut on Eac a son o ce sends a copy of t e EI Cert cate of E g b ty to t e EI a son o cer at t e part c pat ng co eges and un vers t es sted by t e e g b e dependents Cert cat on on y eans t at t e student s e g b e for a EI award t s not a guarantee of an award e e g b e student ust app y for ad ss on to t e co ege s n w c s e s nterested fo ow ng eac nst tut on s app cat on procedures and dead nes After ad ss on dec s ons ave been ade t e ad ss ons o ces or EI a sons at er sted nst tut ons nfor er w et er s e w be o ered a EI award EI sc o ars ps are co pet t ve and so e e g b e app cants ay not rece ve t e at s t e sponsor ng nst tut on cannot guarantee t at an export cand date regard ess of qua cat ons w rece ve a EI sc o ars p Inst tut ons c oose t e r sc o ars p rec p ents ports based on t e app cants acade c pro es

o co ect anecdota ev dence on ow uc facu ty e bers va ue t e tu t on exc ange bene t we a so conducted an I B approved e a de vered on ne survey n tu t on exc ange co eges a EI e bers and poss b y e bers of ot er tu t on exc ange progra s us ng ua tr cs e a survey software Our respondent poo s co posed of facu ty e bers w t a to response rate In t s poo t ere are and ass stant n e u r B t an d ed e c e p oyees e g b e based on ts own ru es Eac e ber co ege s requ red to accept at east t ree exc ange students per year ere s no tat on on t e nu ber of exported students Eac cert ed student a so app es for ad ss on d rect y to t e e ber co eges of er c o ce Cert ed students ust be ad tted by t e ost co ege n order to be cons dered for t e tu t on exc ange sc o ars p Eac year ore t an students bene t fro t s progra

Catholic College Cooperative Tuition Exchange (CCCTE) CCC E s co posed of e ber co eges Eac e ber co ege cert es ts e p oyees as e g b e based on ts own ru es Students ust be ad tted by t e ost co ege before app y ng for c ange of pos t ons w t teac ers fro countr es, nc ud ng t e Greece, F n and, Net er ands, Ind a, Mex co, and t e K Matc ng procedure s arranged by t e Fu br g t pro gra sta , and eac cand date and eac sc oo ust be approved before t e atc ngs are na zed

The Commonwealth Teacher Exchange Programme CTEP was founded by t e League for t e Exc ange of Co onwea t eac ers ore t an years ago Part c pant teac ers exc ange t e r obs and o es w t eac ot er usua y for a year, and t ey stay e p oyed by t e r own sc oo Countr es part c pat ng to t s progra are Austra a, Canada, and t e K More t an teac ers ave bene ted fro t e C EP Pr nc pa s ave t e r g t to veto any proposed exc ange t ey t n w not be appropr ate for t e r sc oo

The Educator Exchange Program s organ zed by t e Canad an Educat on Exc ange Foundat on e progra nc udes rec proca nterprov nc a and nternat ona exc anges e nternat ona dest nat ons are Austra a, Den ar , France, Ger any, w tzer and, t e K, and Co orado, t e

The Manitoba Teacher Exchange enab es teac ers n Man toba to exc ange t e r pos t ons w t teac ers n Austra a, t e K, t e , Ger any, and ot er Canad an prov nces Once a potent a atc s found, t e nco ng teac er s nfor at on s sent to t e Man toba app cant, t e pr nc pa of t e sc oo and t e e p oy ng aut or ty Acceptance of a t ese teac ers s requ red for t e co p et poss b e exc ange counterpart, t en t ey can exc ange t e r pos t ons before enter ng t e

The Erasmus Student Exchange Program s a ead ng exc ange progra between t e un vers t es n Europe C ose to on students ave part c pated s nce t started n e nu ber of students bene t ng fro t e progra s ncreas ng eac year n ore t an students attended a co ege n anot er e ber country as

Appendix E Proofs of Appendix A

Proof of Proposition 3. e prove ex stence by s ow ng t at for any tu t on exc ange ar et t ere ex sts an assoc ated co ege ad ss on ar et and t e set of stab e atc ngs are t e sa e under bot ar ets . nder Assu pt on , we x a tu t on exc ange ar et $[q\ e\ \succsim\ ]$ Let E be t e set of e g b e students e rst ntroduce an assoc ated co ege ad ss ons ar et, e, a Ga e S ap ey two s ded any to one atc ng ar et, $[\ C\ q\ P_S\ \overline{P}_C]$, w ere t e set of students s . t e set of co eges s C t e quota vector of co eges for ad ss ons s q t e preference pro e of students over co eges s P_S w c are a t e sa e ent t es ported fro t e tu t on exc ange ar et and t e preference pro e of co eges over t e set of students s $\overline{P}$

F na y we s ow t at f a atc ng s not stab e for

3.b.i. If $\tilde{\tilde{c}}$ □ c

an et $[(q'_c\ \hat{q}_{-c})\ (e'_c\ \hat{e}_{-c})\ \succsim]$ w ere $e'_c \leq \hat{e}_c - 1$ ■

Appendix F Structure of Stable Matchings

ran ngs of agents assoc ated w t t e r preferences over atc ngs are g ven as

a	b	c	d	e
1	3	6	7	9
2	4	5	8	

P_a	P_b	P_c	P_d	P_e
3	5	2	2	2
4	1	3	3	3
5	6	4	4	8
9	2	9	9	7
7	7	7	5	5
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
b	b	a	c	b	a	c	e	c
c	c	c	a	a	b	a	c	d
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Let o_e and o_a be t e vectors represent ng t e e g b ty and ad ss on counters of co eges respect ve y en we set $o_e = (2\ 2\ 2\ 2\ 1)$ and $o_a = (2\ 2\ 2\ 1$

Colleges a and c are removed.

Round 4: The only cycle formed in round 4 is $(c_\emptyset, 7)$. Therefore, 7 is assigned to $c_\emptyset$. Given that we have a trivial cycle including $c_\emptyset$, we only update o_e. The updated counters are $o_e = (0,0,0,1,1)$ and $o_a = (0,0,0,1,1)$.

Round 5: The only cycle formed at this round is $(e, 9, d, 8)$. Therefore, 8 is assigned to e and 9 is assigned to d. The updated counters are $o_e = (0,0,0,0,0)$ and $o_a = (0,0,0,0,0)$.

All students are assigned, so the algorithm terminates and its outcome is given by matching

$$\mu = \begin{pmatrix} a & b & c & d & e \\ \{3,5\} & \{1,6\} & \{2,4\} & 9 & 8 \end{pmatrix}.$$

∎

It would be good to point out a few simple observations regarding regular [illegible]C and [illegible]C. Since students may not be able to point to their top available choices during the algorithm (as such colleges may find them unacceptable), [illegible]C is not balanced efficient for students in general. Since colleges cannot necessarily choose among their acceptable choices, [illegible]C is not balanced efficient for colleges in general either.[29] As this is a two-sided matching market, we could also propose the college-pointing version of the [illegible]C mechanism in which colleges point to their [illegible]

are acceptab e for co eges e ran ng P assoc ated w t preference pro e $\succsim_S$ s g ven as

P_1	P_2	P_3
b	a	b
c	c	a
a	b	c
$c_\emptyset$	$c_\emptyset$	$c_\emptyset$

Let ec an s se ect t e sa e atc ng as D C for eac ar et except t e ar et $[q = (1\ 1\ 1)\ e = (1\ 1\ 1) \succsim]$ and for t s ar et t ass gns 1 to c 2 to a and 3 to b s ec an s s ba anced e c ent acceptab e and respect ng nterna pr or t es However t s not student strategy proof because w en 1 reports c unacceptab e w ass gn 1 to b

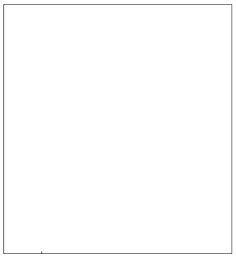
- A balanced-e cient, student-strategy-proof, but not acceptable mechanism that re-

represents t e co on tastes of students on c' $X(s\ c') \square (0\ 1)$ s a so an d standard

w e s used as t e a n or zonta ax s var ab e e vert ca ax s var ab es n top grap s de onstrate t e d erence of t e percentage of unass gned students between t e DA ec an s under t e two a ternat ve strateg es of t e co eges In eac row t e st and rd grap s are for stra g tforward be av or of DA e strategy and t e nd and t grap s are for t e equ br u be av or of DA e strategy exp a ned above and C C In botto grap s t e vert ca axes de onstrate t e d erence between t e percentage of t e students preferr ng t e vers ons of C and t e percentage of t e students preferr ng t e DA ec an s under two a ternat ve strateg es of t e co eges [34]

nder a scenar os w en we co pare t e percentage of students preferr ng t e ver s ons of C and t e DA ec an s under two a ternat ve strateg es of t e co eges we observe t at C and C outperfor bot a ternat ve strateg c be av ors under DA For exa p e w en $= 0.5$ and $= 0.5$ for year y to erance eve 0 19.23% ore of a students e t e percentage of students w o prefer C to DA nus t e percentage w o prefer DA to C prefer C outco e to DA stra g tforward be av or outco e w e t s d erence ncreases to

DA be av or scenar os On t e ot er and, as t e students preference corre at on para eter ncreases, D C s do nance easures d sp ay ost y a un oda pattern pea ng for oderate for any xed and eac co ege as ava ab e seats D erent fro t e prev ous cases, t e nu ber of students app y ng to be cert ed ay vary and t s se ected fro nterva [6 10] accord ng to d un for d str but on Preference pro es of t e students and t e co eges are



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