Blood Allocation with Replacement Donors: A Theory of Multi-unit Exchange with Compatibility-based Preferences*

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Abstract

In 56 developing and developed countries, blood component donations by volunteer non-remunerated donors can only meet less than 50% of the demand. In these countries, blood banks rely on replacement donor programs that provide blood to patients in return for donations made by their relatives or friends. These programs appear to be disorganized, non-transparent, and inefficient. We introduce the design of replacement donor programs and blood allocation schemes as a new application of market design. We introduce optimal blood allocation mechanisms that accommodate fairness, efficiency, and other allocation objectives, together with endogenous exchange rates between received and donated blood units beyond the classical one-for-one exchange. Additionally, the mechanisms provide correct incentives for the patients to bring forward as many replacement donors as possible. This framework and the mechanism class also apply to general applications of multi-unit exchange of indivisible goods with compatibility-based preferences beyond blood allocation with different information problems.

Keywords : Blood transfusion, market design, multi-unit exchange, dichotomous preferences, endogenous pricing

JEL Codes : D47, C78, I19, D82, D78

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1 Introduction

Transfusions are commonly used to treat various medical conditions to replace lost blood or add inadequate blood components. Replacement red blood cells and other blood components such as platelets, plasma, and clotting factors are essential for patients going through certain procedures such as surgery, chemotherapy, and child birth and for patients with trauma and blood diseases[1]. In the US, according to Pfuntner et al. (2013), blood transfusion was the most common procedure performed during hospitalizations in 2011. Even though transfusion is an essential procedure in health care, many patients around the world do not have access to safe blood due to signi cant shortages.

Around the world, the collection and distribution of blood is organized through blood banks where donated blood is processed and stored. Unlike most solid human organs and tissues, blood replenishes after donation and most blood products can be stored for a period of time. Thus, a healthy donor can donate whole blood regularly once in every two months and some components, such as platelets and plasma, more frequently. Di erent compatibility requirements apply for each blood component (see Section 2 for medical and institutional details of blood component transfusion including various compatibility requirements). rf 3-ad351822843o1al.o1ap2050WHO,

The most adequate and reliable supply of blood is through volunteer non-remunerated donors (VNRDs), who mostly donate blood, often repeatedly, through blood drives or other campaigns[2]. These donors provide the safest supply of blood, since the prevalence of blood-borne infections is lowest among this group of donors[3]. According to the World Health Organization (WHO), 79 countries (38 high-income, 33 middle-income, and 8 low-income) collect more than 90% of their blood supply from VNRDs (WHO, 2020). The World Health Assembly resolution WHA63.12 (Sixty-third World Health Assembly, 2010) urges all member states to develop national blood systems based on VNRDs and to work toward the goal of self-su ciency. Despite these warnings, donations by VNRDs remain insu cient to meet the demand for blood and its components in many regions of the world.

Although it seems relatively costless to donate blood, there are severe blood shortages in many developing countries, as well as seasonal shortages in developed countries (Gilcher and McCombs, 2005).[4] Cultural and religious factors create frictions that deter VNRDs, especially in some developing countries. Furthermore, some blood components, such as platelets, have short shelf life, are in high demand, and are more difficult to collect than the others. Thus, shortages of such components occur even in the developed world.

In 56 countries worldwide (9 high-income, 37 middle-income, and 10 low-income), more than 50% of the blood supply is met by *replacement donors* and, in some cases, through paid donors (WHO, 2020). As an effective method to boost blood component reserves, blood banks in many places—including highly populated countries such as India, China, and Brazil—employ official or unofficial *replacement donor programs*. A replacement donor program requires each patient to nominate a number of willing donors, who are typically family members or close friends, to donate in order for the patient to receive transfusion.[5]

Notwithstanding the important role they play in addressing blood shortages, existing replacement donor programs suffer from two major shortcomings.

The first shortcoming is the loss of welfare due to the lack of optimized inventory management based on donor screening and the needs of the blood bank. Although inventory management is often considered among the most important goals for a blood bank, as far as we know, no explicit optimization is pursued in current replacement donor programs to achieve certain policy objectives. In the face of chronic supply shortages, one such natural objective can be to maximize the allocated blood volume using the correct set of replacement donors.

The second shortcoming is that replacement donor programs generally operate on

[4]There are often shortages of type O red blood cells in the US in the early winter and midsummer months. Outside of seasonal factors, blood shortages can often frequently occur during catastrophic events such as earthquakes or pandemics. For example, during the recent COVID-19 pandemic, blood components have had shortages in the US (American Red Cross, 2020a).

[5]Within the medical community, there is an ongoing debate about the stance of the WHO regarding VNRDs being the safest blood supply. There has been considerable evidence suggesting that the blood collected through replacement donors is as safe as VNRDs. It is also argued that the motivations of the two types of donations are similarly altruistic, and the distinction between them from an ethical perspective is not clear cut. Allain and Sibinga (2016) provide an excellent survey of these views, empirical evidence, and references. In addition, there are significant economic and cultural reasons for the predominance of decentralized and often hospital-based replacement systems in many developing countries. Such a system is much less costly (Bates et al., 2007), favors intra-group solidarity, and is culturally more consistent with the presence of strong family or community bonds (Haddad et al., 2018; Kyeyune-Byabazaire and Hume, 2019).

xed exchange rates between units (of blood) received by the patient and units supplied by the patient's donors, which creates issues of e ciency, fairness, and ethics. Certain patients may not be able to recruit the required number of donors that they are obliged to provide, making it di cult to receive blood. The rules of replacement donor programs are sometimes bent arbitrarily in favor of such patients, or such patients pay third parties to assume the role of their replacement donors creating black markets. Additionally, around the world, replacement donor programs appear to be highly non-transparent in their blood allocation operations. It is di cult to nd existing guidelines that govern these processes (see Section 2 for institutional details of how real-life replacement donor programs function). Even in the absence of these problems, a xed exchange rate regime limits the scope of admissible exchanges and allocations.

In this paper, we introduce blood allocation with VNRDs and replacement donors spondence. We view the design of feasible schedule correspondences as an important policy variable and novelty in the paper.

Then we propose and study a general class of **optimal mechanisms**. Each optimal mechanism is represented by the maximization of an *additively responsive aggregate* preference relation over schedule profiles of the patients, subject to feasibility constraints designated by the feasible schedule correspondence of each patient, as well as market clearing and blood-type compatibility conditions (see Section 4). This class includes practical mechanisms that fulfill the blood bank's various allocation and inventory management objectives, such as *sequential targeting mechanisms* (that maximize the amount of blood received or minimize the amount of blood supplied by each target patient group in a sequential manner) and *weighted maximal mechanisms* (that maximize the difference between a weighted sum of the amounts received by the patients and a weighted sum of the amounts supplied). Optimal mechanisms also nest all previously studied mechanisms for the multi-unit exchange of indivisible goods with compatibility-based monotonic preferences as special cases (see Section 6).

The optimal mechanisms together with the feasible schedule correspondences overcome the two shortcomings of current replacement donor programs outlined above.

First, they address the lack of optimization based on donor screening. In particular, the optimal mechanisms are efficient for patients under basic alignment conditions of the aggregate preference relation over schedule profiles with patients' preferences (Remark 1). They are also *donor monotonic*, i.e., providing a larger set of donors does not reduce the amount of blood the patient receives, under three natural restrictions on the feasible schedule correspondences (Theorem 2): every feasible schedule set satisfies a discrete convexity notion, L(attice)-convexity; if a patient receives an extra unit of blood, then it is also feasible that an additional donor of hers can be asked to donate, if needed; and the feasible schedule set becomes more favorable for the patient as her donor set expands. Among these conditions, L-convexity plays an important role, which also guarantees that the outcome of a weighted maximal mechanism can be found in polynomial time (see Appendix C.2 in Supplemental Material). Achieving donor monotonicity is particularly important in this context as it helps align patients' individual incentives with the blood bank's objective of increasing blood transfusion. We show that optimal mechanisms satisfy a stronger incentive compatibility notion when the last restriction on the feasible schedule correspondences is strengthened (Theorem 3).[6]

[6] We also provide comparative static analysis for changes in feasible schedule correspondences (The-

Second, the innovation of feasible schedule correspondences allows for various exchange rates between units received and supplied, while optimal mechanisms determine endogenously these exchange rates. This property helps rectify the shortcoming caused by a fixed exchange rate in current programs, as these feasible schedule correspondences can be tailored fairly for patients who can intrinsically recruit fewer donors, or for different medical conditions, which help prevent black markets. As a result, our approach provides a framework to assess and improve the effectiveness of the existing replacement donor programs, and makes it possible to offer rigor and transparency to their organization. Toward this goal, we provide concrete policy designs and implementation proposals (see Section 5). We also conduct simulations to show the possible gains from our design. Using the blood-type distribution in India and under a set of realistic parameters, a sequential targeting mechanism under flexible exchange rates leads to 19%-28% more transfusions than the same mechanism under the one-for-one exchange rate, which in turn leads to 164% to 3% more transfusions than an emulation of current replacement donor practices.

Unlike the living-donor organ exchanges that have attracted much attention in the last two decades in both the market design literature and practice, blood allocation involves multi-unit demand and supply.[7] Moreover, many other factors make this market design problem theoretically and practically different from the analysis and functioning of solid organ exchanges. These include differences in the compatibility requirements for different blood components, the possibility of endogenous and non-unit exchange rates between blood received and supplied, the non-simultaneity between donation and transfusion, and the possibility to store blood components.

Our model and theoretical results are independent of the particular background of blood allocation and can readily be applied to other contexts with a subset of similar features within the framework of multi-unit exchange of indivisible goods with compatibility-based monotonic preferences in units consumed. Although compatibility is verifiable in blood allocation, there can be other contexts where this is private information for each agent. Some such applications studied in the literature include shift exchanges among the workers in a company (Manjunath and Westkamp, 2021) and time banks and favor exchanges (Andersson et al., 2021). We show that optimal mechanisms are weakly

orem 4).

[7]Notable exceptions to unit-demand organ exchanges are living dual-donor lobar lung transplantation,

strategy-proof under our baseline assumptions: no agent receives more compatible units by misreporting her compatibility relation and/or under-reporting her endowment set.[8] Under more stringent conditions, we show that they are fully strategy-proof. Thus, our mechanisms and incentive results substantially generalize and subsume previous ones under compatibility-based preferences. Moreover, as far as we are aware, all previous exchange mechanisms in the literature use the exogenous one-for-one exchange rate. As an important theoretical contribution, we overcome this limitation and introduce endogenous pricing of units while maintaining the good incentive properties of the mechanisms (see Section 6 for more on this and other related literature).

2 Background

2.1 Main Blood Components and Compatibility

There are different transfusion protocols for different blood components, and the medical practices also vary across different regions of the world. We mainly focus on the three most-transfused blood components—red blood cells, platelets, and plasma—as well as whole blood, and provide a brief account starting with a general rule of thumb for compatibility requirements.

Blood-type compatibility plays an important role for the feasibility of transfusion. There are more than 300 human blood groups. Two of them are the most important in clinical practices. The first one, the ABO blood group system, is the most commonly known. A person's ABO blood type is determined by the presence of A or B antigens in her blood cells: her type may be O (if she has neither antigen), A (has only the A antigen), B (has only the B antigen), or AB (has both antigens). Each person has preformed antibodies in her plasma against every non-existent antigen. Antibodies against an antigen attack blood cells that carry this antigen, which can cause potentially fatal hemolysis.

Therefore, any transfusion including a significant amount of donor cells, by rule of thumb, should respect *ABO-cellular compatibility*: O blood-type cells can be donated to all, A blood-type cells can be donated to A and AB blood-type patients, B blood-type cells can be donated to B and AB blood-type patients, and AB blood-type cells can only be donated to AB blood-type patients.

[8] We extend our analysis to this general domain in Appendix B in Supplemental Material and consider the incentives to truthfully reveal compatibility relation as well as endowment. The proof of our main result, Theorem 2, generally applies to prove this new result and only certain points need to be modified as noted in this appendix.

On the other hand, any transfusion including a significant amount of donor plasma, which carries the donor's pre-formed antibodies, by rule of thumb, should respect ABO-plasma compatibility. AB blood-type plasma can be donated to all as it does not contain any antibodies, A blood-type plasma can be donated to A and O blood-type patients, B blood-type plasma can be donated to B and O blood-type patients, and O blood-type plasma can only be donated to O blood-type patients as it contains antibodies against both antigens.

The second crucial blood group system is Rh. The most clinically important Rh antigen is D. Its existence and non-existence correspond to Rh D+ type and Rh D− type respectively. Antibodies to the Rh D antigen can only develop on an Rh D− person after being exposed to Rh D+ red blood cells. Hence, the compatibility requirement is to avoid the transfusion of Rh D+ red blood cells to an Rh D− patient, due to the risk of hemolytic reactions.

Most blood components are packed with others in solutions. Thus, depending on the amount of these components, different practices are followed for the compatibility of the pack with the patient.

Next, we turn our focus to specific blood components.

Red Blood Cells: Red blood cells carry oxygen from the lungs to all parts of the body and are the most commonly transfused blood components. Red blood cell transfusion—the de-facto modern day replacement for the older whole blood transfusion therapy—is mostly used for patients with anemia due to cancer, blood diseases, and other causes, followed by surgical patients. Whole blood is still transfused in some low-income countries. For other countries, this is only occasionally performed in emergencies for patients with massive blood loss due to trauma, surgeries, etc. A person donates one unit (about a pint) of whole blood each time and she has to wait at least eight weeks between donations. Each unit of red blood cells is prepared from one unit of donated whole blood by removing plasma and adding preservative solutions, and can be stored for about 42 days.

ABO-identical and Rh D-compatible transfusion is generally practiced for whole blood transfusion.[9] For red blood cells, ABO-cellular compatible and Rh D-compatible transfusion is all that is needed in theory. However, as red blood cell packs usually carry some amount of donor plasma, ABO-identical (and Rh D-compatible) transfusion is often re-

[9] An exception is that type O Rh D− blood is often transfused in emergencies to patients with other or unknown blood types. For this reason it is also dubbed as the *global-donor* blood type.

quired.

Eight blood types are relevant for red blood cell or whole blood transfusion. However, in some populations, such as those in Asia, Rh D- is so rare that there are effectively only four blood types.[10]

Platelets : These are tiny cells in the blood that form clots and stop bleeding. Platelet transfusions are mostly given to prevent or treat bleeding in patients with thrombocytopenia or abnormal platelet function, such as those undergoing chemotherapy or receiving a bone marrow transplant. McCullough (2010) states that the use of platelets has increased more than other blood components in the last 15 years. According to Red Cross of America, every 15 seconds someone needs platelets (American Red Cross, 2020b). However, due to their storage requirement at room temperature, platelets have a much shorter shelf life than most other blood components: in most countries they can only be stored between four and seven days (Cid et al., 2013). As a result, platelets are in frequent shortages even in developed countries.

One unit of platelets can be prepared from 4-6 units of pooled whole blood, or obtained from a single donation through the technique of apheresis, which only takes platelets out of the donor's blood, leaving the other components in the blood stream. The whole process takes approximately three hours and a person can donate platelets in this way once a week, up to 24 times a year.[11] In addition to the efficiency in the production process, apheresis platelets are also safer to the patients due to the minimal donor exposure. Hence, it has become an increasingly common practice to give apheresis platelets, instead of whole-blood-derived platelets. In 2017, only 2.4% of the total transfused platelet units in the US were derived from whole blood (Jones et al., 2020).[12]

For platelets, the compatibility practices vary significantly among different institutions and countries. As platelets (weakly) express the ABO antigens and they are sus-

Finally, as the Rh D antigen is not present on platelets, Rh D compatibility is usually not required (for example, see Cid et al., 2013).

Plasma: It is the non-cellular, protein- and antibody-rich liquid component of blood. The plasma used in everyday transfusion is usually fresh frozen plasma Plasma transfusion is often utilized by patients with liver failure, heart surgery, severe infections, and serious burns. One unit of fresh frozen plasma can be prepared from one unit of whole blood after removing the blood cells. Alternatively, a person can donate up to three units through apheresis, which keeps other blood components in her blood stream and only extracts plasma. Fresh frozen plasma has the longest shelf life among the three main blood components: it can be stored for about a year. Its transfusion follows ABO-plasma compatibility, without regard to Rh D compatibility (as Rh D antibodies only form after exposure to the Rh D antigen and are not pre-formed).

Convalescent plasma, the antibody-rich plasma of a patient recovering from an infectious disease with no other known cure, such as Ebola and most recently COVID-19, is commonly used to treat patients or to produce drugs against the disease. It can also be considered as a type of fresh frozen plasma.

In addition to plasma used for transfusion, plasma derivatives (such as albumin, coagulation factors, and immunoglobulins) manufactured from \source plasma" in fractionation centers are used in the treatment of various conditions. Unlike the blood used for transfusion, source plasma is commonly collected from paid donors in many countries.[13]

2.2 Blood Demand of a Patient

The amount of a blood component needed to treat each medical condition is idiosyncratic. For example, Collins et al. (2015) report that, at a tertiary referral center in the US, the average amount of red blood cell units used per surgery is close to 3.5 units and this amount has a high variance due to various patient conditions.

Besides the idiosyncratic demand, there is usually a range of units where each amount in the range can be transfused to a given patient. However, receiving more units is generally better under various outcome or preference metrics. We give three general examples of patient demand that have this common thread.

blood needs to be tested and processed rst), the blood bank is used as an intermediary.

Blood banks work with hospitals and blood centers. Hospitals relay the needs of patients to the blood banks, while the blood banks and blood centers collect donations from VNRDs and replacement donors. Hospitals are often required to maintain a small inventory of their own (for example, see Delhi State Health Mission, 2016).

Although replacement donor programs are very common and o cially acknowledged in many countries, maybe surprisingly, it is di cult to nd their exact institutional details. The most common practice in current replacement donor programs worldwide is that the blood bank announces, either o cially or uno cially, a preset exchange rate between the units of blood received and supplied, often irrespective of the blood type sought or donated. Blood banks provide blood to patients exclusively based on these rates. Among these, the one-for-one exchange rate, i.e., one unit replacement per unit received, is most common around the world.

We also give some examples of other policies practiced. Although China banned the replacement donor programs in 2018, they are still used in several cities during shortages, especially for platelets (She, 2020). Di erent policies have been in place. In most cities, including Beijing, the exchange rate is one-for-one. As reported by She (2020), in Xi'an, during periods of shortages, a patient has the priority of receiving three units of blood for every unit she has donated before, and she has the priority of receiving one unit for every unit her replacement donors donate now. According to Chen (2012), in Guangzhou, there is not necessarily a xed relation between the amount received and supplied. Moreover, in some regions there are restrictions on the blood types of replacement donations. As an extreme case, the blood type of a replacement donor must be identical to that of the patient in Jiangsu. While such a restriction is relatively rare for whole blood donations, it is not uncommon for replacement platelet donations throughout the country.

India has the largest o cial replacement donor programs in the world after Pakistan. In Delhi, regardless of the amount of blood she needs, the patient is required to bring forward one replacement donor, unless the intervention needed is an emergency surgery (Delhi State Health Mission, 2016).

In Cameroon and Congo, the exchange rate has been two replacement units per unit received, as almost 25% of the don-269(d)1(onthe)-27(h)l

The exchange rate is fixed at one-for-one; however, it is not as strictly enforced.[16]

2.4 Institutional Constraints

The feasibility of blood transfusion primarily depends on the blood type compatibility. Therefore, replacement donor programs operate on the premise of exchange of willing donors for compatible blood received by the paired patient. This is similar in principle to organ exchanges with the first-order difference that there is not yet an optimized central clearinghouse for replacement donors. There are a number of other important institutional differences. To begin with, the logistical constraints of blood donation are negligible compared to those in organ transplantations. The blood donation process takes only a few hours and its effects wear off relatively quickly. On the other hand, organ transplantations carry risks and require careful planning weeks before and after the operations. Once extracted, blood components can be stored for a certain period of time, which can facilitate the designer's choice of optimal timing of assignments. Moreover, many blood banks and hospitals often operate in coordination, making it possible to obtain the necessary blood units from neighboring facilities. These lead to the observation that in blood allocation with replacement donors, the possibility of a donor reneging is not as much of a concern as in organ exchanges.[17]

The logistical ease and flexibility in blood allocation have led to different and innovative incentivization schemes to promote blood donation. The assignment of voucher credits has been a popular approach in practice. For example, blood assurance programs in the US guarantee each VNRD or her tax-code dependents exactly the same amount of blood donated in the event of a future need.[18] Similar programs have also been traditionally implemented in China. Recently, Kominers et al. (2020) proposed a similar incentive scheme for COVID-19 convalescent plasma donation.[19] Replacement donor programs differ from these proposals, as we are considering the improvement of existing programs that usually do not have many voucher or memory features, nor the pay-it-backward constraints.

3 The Model

We consider the market for a single blood component, which we simply refer [redacted] blood.[20] Let I be a set of patients and B be the set of blood types.[21] Each X 2 [redacted] denotes a speci c blood type used in compatibility requirements. Each patient i 2 I ha[redacted] type $_i$ 2 B blood and needs a maximum of $\bar{n}_i$ 2 Z_{++} units of blood. For each X 2 B[redacted] $C(X)$ B, $C(X)$ 6= ;, is the set of blood types compatible with a type X patient. Eac[redacted] patient i also has a (possibly empty) set of willing replacement donors D_i such tha[redacted] each donor d 2 D_i can provide one unit of type $_d$ 2 B blood. Let D_i be the collectio[redacted] of all possible donor sets that a patient i 2 I can bring forward. Assume that if D_i 2 D[redacted] and D_i^0 D_i, then D_i^0 2 D_i. Let $_I$ = ($_i$)$_{i2I}$, $_D$

may be set to zero during severe shortages.

Since each patient demands and (possibly) supplies blood through her replacement donors, we impose restrictions on the relationship between the amount of blood received and the amount of blood supplied. A *schedule* is a pair of non-negative integers (r, s), where r denotes the amount of compatible blood received and s denotes the amount of blood supplied. For every patient $i \in I$, her feasible schedule correspondence S_i assigns a non-empty set of schedules $S_i(D_i$

every $D_i \in \mathcal{D}_i$,

$$S_i(D_i) = \begin{cases} \{(0,0)\} & \text{if } |D_i| < 2\underline{n}_i \\ \left\{(r,s) \in \mathbb{Z}^2_+ : s = 2r \text{ and } \underline{n}_i \leq r \leq \min\left\{\overline{n}_i, |D_i|/2\right\}\right\} & \text{otherwise} \end{cases}.$$

- Xi'an, China policy: A patient is guaranteed three units for each unit she has donated before, and the exchange rate is one-for-one beyond this guarantee (She, 2020). Let $x_i \in \mathbb{Z}_+$ be the amount of previous donations from the patient.[24] Then, her feasible schedule correspondence is as follows.

  If $\overline{n}_i \leq 3x_i$, then for every $D_i \in \mathcal{D}_i$,

  $$S_i(D_i) = \{(\overline{n}_i, 0)\}.$$

  If $\overline{n}_i > 3x_i$, then for every $D_i \in \mathcal{D}_i$,

  $$S_i(D_i) = \left\{(r,s) \in \mathbb{Z}^2_+ : s = r - \underline{n}_i \text{ and } \underline{n}_i \leq r \leq \min\{|D_i| + \underline{n}_i, \overline{n}_i\}\right\},$$

  where $\underline{n}_i = 3x_i$.

- Jiangsu, China policy: The standard one-for-one policy is used with the restriction that the type of the blood supplied must be identical to the type of the patient (Chen, 2012): for every $D_i \in \mathcal{D}_i$, if $|\{d \in D_i : \tau_d = \tau_i\}| < \underline{n}_i$, then

  $$S_i(D_i) = \{(0,0)\};$$

  and otherwise,

  $$S_i(D_i) = \left\{(r,s) \in \mathbb{Z}^2_+ : s = r,\ \underline{n}_i \leq r \leq \min\{\overline{n}_i, |\{d \in D_i : \tau_d = \tau_i\}|\}\right\}.$$

  This is akin to no exchange (autarky) treatment. ■

A blood allocation problem with replacement donors is denoted as $P = \langle I, \tau_I, \overline{n}, D, \tau_D, v, \underline{n}, S\rangle$. The inventory vector v, minimum guarantees $\underline{n}$, and feasible schedule correspondences S are interrelated and can all be considered as policy levers.[25] We fix every component of a problem except D.[26] Then a problem is simply denoted as a donor profile D.

Given a problem $D \in \mathcal{D}$, an allocation μ consists of non-negative integers $\mu_X(i)$ for each $i \in I$ and $X \in C(\tau_i)$, and $\mu(d) \in \{0,1\}$ for each $d \in \bigcup_{i \in I} D_i$ such that

[24] Assume that x_i is exogenous to the problem, and the patient has not used the credits received from the previous donations in a replacement donor program.

[25] The vector v can be interpreted as the minimum required inventory level to be kept in stock. This is mostly ensured through a blood exchange program among blood banks, which is commonly practiced (for example, see AABB, 2020).

[26] Without loss of generality, we use this notation for brevity, assuming τ_D is determined once D is given. Moreover, in Section 4.3, we discuss the effect of changing a patient's feasible schedule correspondence.

1. for every $X \in B$, $\sum_{i \in I : X \in C(\tau_i)} \alpha_X(i) \leq v_X + \sum_{d \in \cup_{i \in I} D_i : \tau_d = X} \beta(d)$,
2. for every $i \in I$, $\left(\alpha(i); \sum_{d \in D_i} \beta(d)\right) \in S_i(D_i)$, where $\alpha(i) = \sum_{X \in C(\tau_i)} \alpha_X(i)$.

In an allocation, the patients only receive blood that is medically compatible with them. An allocation specifies the amount of blood of each compatible type that a patient receives, as well as which of her donors donate. The first condition in the definition makes sure that, for each blood type, the allocated blood is not more than the sum of the existing blood in the blood bank and the collected blood from the patients' donors.

$w_j(\ ^0) P_j\ w_j(\ )$ for some $j\ 2\ I$.

A mechanism is a function f that maps each problem $D\ 2\ D$ to an allocation $f(D)\ 2\ A(D)$. A mechanism f is e cient if for every $D\ 2\ D$, $f(D)$ is e cient.

We consider the patients' incentives for bringing forward their donors. We introduce two notions of incentive compatibility, one weak and one strong, where the latter one coincides with strategy-proofnessiderery

Formally, the mechanism designer has a complete, transitive, and antisymmetric aggregate preference relation   over all schedule profiles in the set W. The asymmetric component of   is denoted as . A mechanism f is induced by the aggregate preference relation   if for every problem $D \in \mathcal{D}$,

$$f(D) \in \quad \in A(D) : w(\ ) \quad w(\ '); \forall \ ' \in A(D) \ :$$

We define two additional conditions on the mechanism designer's preferences. First, the aggregate preference relation   is aligned with patients' preferences if for every two schedule profiles w and w' such that $w_i \; R_i \; w'_i$ for all $i \in I$, we have $w \quad w'$. That is, if every patient weakly prefers w to w', then the mechanism designer also weakly prefers w to w'. Second, we say $w \in W$ is a basic schedule profile if $w \in \{0;1\}^{2|I|}$, i.e., each element of the vector is either 0 or 1. In a basic schedule profile, there is a subset of patients who each receive a single unit of blood, and a subset of patients who each supply a single unit. The aggregate preference relation   is (additively) responsive to the designates for each subset N_k whether a maximization or minimization target will be achieved.

Maximization, denoted by (k) = max, means that the total amount of blood received by the patients in N_k is maximized given that the previous objectives are already satis ed.

Minimization, denoted by (k) = min, means that the total amount of blood supplied by the (donors of) patients in N_k

that $i \in N_{k'}$ and $\tau(k') = \max$. That is, if we are going to minimize the blood supplied by a group of patients, then for each of those patients, we should have maximized the blood received by some group that includes her at an earlier step.

The first condition guarantees that the outcome allocations of the procedure are welfare equivalent: we use the last $\bar{k} - 2$ targets as tie breakers among the patients, in case the previous targets lead to a multiplicity of allocations in terms of welfare levels. As the preferences of the patients are lexicographic in receiving more blood and then supplying less blood, the second condition will ensure the efficiency of sequential targeting.

A sequential targeting mechanism φ is defined through the above procedure with respect to a sequence of target sets $\{N_k\}_{k=1}^{\bar{k}}$ and a target function τ that satisfy the above two conditions: it chooses an allocation from the outcome set of the procedure, $A_{\bar{k}}$, executed for each problem $D \in \mathcal{D}$.

A sequential targeting mechanism is induced by a lexicographic preference relation of the mechanism designer, such that given any two schedule profiles, he prefers the one in which the first target set receives more blood; when the amounts of blood received by the first target set are the same, he prefers the one in which the second target set receives more blood (supplies less blood) if the target is maximization (minimization), and so on.

Theorem 1. Every sequential targeting mechanism is an optimal mechanism.

Different target sets and target functions induce different sequential targeting mechanisms. In practice, since blood transfusion is one of the most common medical procedures, the patients requesting blood can be highly heterogenous. Target sets can be designed were examined by Manjunath and Westkamp (2021) and Andersson et al. (2021), respectively, in similar setups. In our context, these two classes of mechanisms are also more broadly defined due to the general specification of feasible schedules.

In a priority mechanism , the patients are processed one at a time using a priority order. Let $|I| = n$ and list the patients in this order as $i_1, i_2, \ldots, i_n$: the mechanism first maximizes the welfare of i_1; then, among all allocations that achieve this goal, it maximizes the welfare of i_2, and so on. Formally, the target sets are singletons such that $N_{2k-1} = N_{2k} = \{i_k\}$ for every $k \in \{1, \ldots, n\}$. The target function is defined as $(2k-1) = \max$ and $(2k) = \min$ for every $k \in \{1, \ldots, n\}$.[30]

In a maximal mechanism with priority tie-breakers , the total amount of blood received by all the patients is maximized, then the total amount of blood donated by all replacement donors is minimized. List the patients as $i_1, i_2, \ldots, i_n$ using a priority tie-breaker. Then among all total welfare maximizing allocations, the welfare of i_1 is maximized. Subject to this goal being satisfied, the welfare of i_2 is maximized, and so on. Formally, the first two target sets are the set of all patients: $N_1 = N_2 = I$. The remaining target sets are singletons such that $N_{2k-1} = N_{2k} = \{i_{k-1}\}$ for every $k \in \{2, \ldots, n+1\}$. The target function is defined as $(2k-1) = \max$ and $(2k) = \min$ for every $k \in \{1, \ldots, n+1\}$.[31]

Another interesting subclass of optimal mechanisms are weighted maximal mecha mechanism that is induced by the aggregate preference relation $\trianglerighteq$, defined as follows. For any two schedule profiles w and w' such that $w \neq w'$, let $w \vartriangleright w'$ if $O(w) > O(w')$, or, $O(w) = O(w')$ and there exists $k \in \{1, \dots, |I|\}$ such that $w_k \, P_k \, w'_k$ and $w_\ell \, I_\ell \, w'_\ell$ for all $\ell < k$. In addition, let $w \trianglerighteq w$ for any schedule profile w. It is straightforward to check that $\trianglerighteq$ is complete, transitive, antisymmetric, and responsive. Moreover, to ensure that it is aligned with the patients' preferences, we assume that for every $i \in I$ and $D_i \in \mathcal{D}_i$, $W^r(i) \trianglerighteq W^s(i)|D_i|$.[32] Then, a weighted maximal mechanism is an optimal mechanism. Moreover, the class of weighted maximal mechanisms subsumes the sequential targeting mechanisms (see Appendix C.1 in Supplemental Material).[33]

4.1 Donor Monotonicity

In this subsection, we explore the incentives faced by patients in bringing forward their full sets of donors to the blood bank.

For a general profile of feasible schedule correspondences $\mathcal{S}$, the optimal mechanisms may not be incentive compatible even in the donor monotonicity sense. We will state regularity conditions on the feasible schedule correspondences that many real-life policies—such as one-for-one exchange—obey.

We make three assumptions which ensure that the optimal mechanisms are donor monotonic. They all have natural explanations. The first one is about the convexity of a feasible schedule set for a given set of donors. Generally, a set $S \subseteq \mathbb{Z}^2_+$ is L-convex (where L stands for lattice) if for every $x, y \in S$, we have

$$\left(\left\lceil \frac{x+y}{2} \right\rceil, \left\lfloor \frac{x+y}{2} \right\rfloor \right) \in S.$$

L-convexity is one of the two most used generalizations of convexity to discrete domains.[34]

Assumption 1 (L-convexity). *The feasible schedule set $S_i(D_i)$ is L-convex for every $i \in I$ and $D_i \in \mathcal{D}_i$.*

Figure 1 provides a geometric illustration with three examples of L-convex feasible schedule sets. Assumption 1 also guarantees that an outcome allocation of a weighted maximal mechanism can be found in polynomial time, as shown in Appendix C.2 in Supplemental Material.

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[32] This assumption implies that for any $w, w' \in W$ with $w_i \, R_i \, w'_i$ for all $i \in I$, we have $O(w) \geq O(w')$.

[33] It is also worth mentioning that given a general optimal mechanism induced by an aggregate preference relation $\trianglerighteq$, there may not exist a linear utility function that represents $\trianglerighteq$, and thus the class of

Figure 1: Illustration of Assumption 1, L-convexity. The feasible schedule set $S_i(D_i)$ is the integral points of a convex polygon with integral corners and at most six edges of slopes 1, 0, or 1 . The best schedule $\overline{S}$ and the worst schedule $\underline{S}$ are also marked in each graph.

A special case that satis es Assumption 1 is the classical one-for-one exchange rate between the blood received and supplied, as depicted in Figure 2.

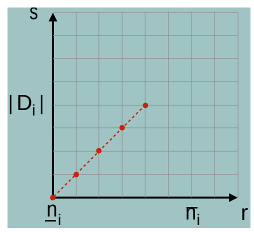

Figure 2: An L-convex feasible schedule set induced by the one-for-one exchange rate policy. In this example, we assume $\overline{n}_i >$ ifd [(i)]TJ/F11 10.9091335346.857 1.636 Td [(>)]TJ/F56 10.9091 Tf 11.96 9

L-convexity and feasibility of positive price are independent. For example, the two-for-one exchange rate policy, i.e., two units supplied for each unit received, satisfies feasibility of positive price but not L-convexity;[35] the second feasible schedule set in Figure 1 violates feasibility of positive price as it has a "flat top" at $s = 5 < |D_i|$ and a "flat bottom" at $s = 1 > 0$, while it is L-convex. The other sets in this figure satisfy feasibility of positive price, although the third one has a "flat top." This is because the "flat top" occurs at the maximum possible supply $s = |D_i|$.

Before presenting the final assumption, we introduce a concept regarding the ranking of schedule sets for the patients, which will also be useful in the comparative static analysis in Section 4.3. Given a patient $i \in I$, a donor set $D_i \in \mathcal{D}_i$ and two sets $S, S' \subseteq W_i$, we say S is weakly more favorable than S' at D_i if the following holds:

- if $(r, s) \in S'$ and $r \geq \underline{n}_i$, then there exists $s' \leq s$ such that $(r, s') \in S$; and
- if $(r, s) \in S$, $s \leq |D_i|$ and $(r, s') \in S'$, then there exists $s'' \geq s$ such that $(r, s'') \in S'$.

When S and S' are schedule sets for a patient, S is weakly more favorable than S' at her donor set if (i) for any schedule in S' such that the amount received is at least the minimum guarantee, there is a schedule in S where the patient receives the same amount by supplying weakly less blood, and (ii) for any schedule in S such that the amount supplied does not exceed the number of donors, whenever there is a schedule in S' where she receives the same amount of blood, there is a schedule in S' where she receives this amount by supplying weakly more blood.

Using this concept, we make the following assumption regarding the relation between feasible schedule sets when a patient reports different sets of donors.

Assumption 3 (Non-diminishing favorability in donors)**.** For every patient $i \in I$ and donor sets $D_i, D_i' \in \mathcal{D}_i$ such that $D_i' \subseteq D_i$, $S_i(D_i)$ is weakly more favorable than $S_i(D_i')$ at D_i'.

Favorability manifests itself geometrically as $S_i(D_i)$ being an expansion of $S_i(D_i')$ in the direction of receiving more blood, and/or a downward shift of $S_i(D_i')$.[36] In addition to Assumptions 1 and 2, the one-for-one exchange rate policy satisfies non-diminishing favorability in donors as well, since the feasible $(r, s') \in S$

number of donors increases. In Figures 3 and 4, we give two examples involving endogenously determined exchange rates to further illustrate the implications of Assumption 3 in conjunction with Assumptions 1 and 2.

Figure 3: An illustration of a feasible schedule correspondence S_i satisfying Assumptions 1, 2, and 3. This particular policy relies only on the number of donors brought forward $|D_i|$ but not other speci cs of the donor set. The rst four graphs illustrate $S_i(D_i)$ for $|D_i| = 0; \ldots; 5$, while the last graph shows how the feasible schedule set changes as the number of donors increases.

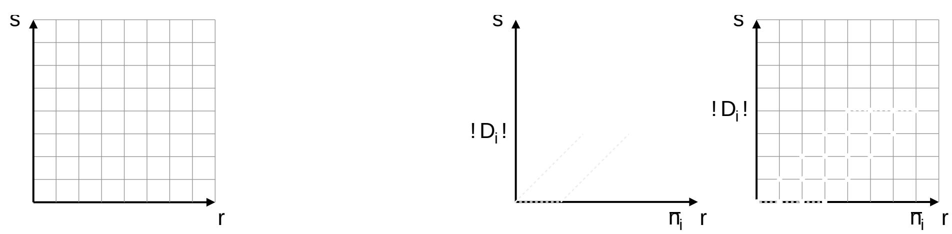

Figure 4: An illustration of a feasible schedule correspondence S_i satisfying Assumptions 1, 2, and 3. The rst four graphs illustrate $S_i(D_i)$ for $|D_i| = 1; \ldots; 4$. The last graph shows how the feasible schedule set changes as the number of donors increases.

The main result of this section is as follows:

Theorem 2. Under Assumptions 1, 2, and 3, every optimal mechanism is donor monotonic.[37]

The proof of this result is substantially involved and we relegate it to Appendix A. We give a sketch of the proof here.

Proof Sketch. We rst de ne an auxiliary matching market that is isomorphic to the original problem, which we refer to as an extended problem. In this market, the blood bank is represented as a pseudo-patient and its inventory is represented by pseudo-donors paired with it. For each blood type, we also introdml941ueachyhth with dummy donors so that, without loss of generality, we can focus on the simple case where any patient cannot receive blood from her own compatible donors. In such an extended

Finally, Lemma 4 states that the optimal rules are donor monotonic. We proceed by contradiction. Let F be an optimal rule, $\hat{D}$ be an extended problem, and $\hat{D}'$ be the extended problem induced by patient i concealing a donor. Suppose that patient i receives more blood under the matching $F(\hat{D}')$ than under the matching $F(\hat{D})$. By Lemma 3, there is a cycle or a chain C from $F(\hat{D})$ to $F(\hat{D}')$. Then, $F(\hat{D}) + C$ is a matching for $\hat{D}$ and $F(\hat{D}')$ C is a matching for

Assumption 4. For every patient $i \in I$ and donor sets $D_i, D_i' \in \mathcal{D}_i$ such that $D_i' \subseteq D_i$, we have

- if $(r,s) \in S_i(D_i')$ and $r \geq \underline{n}_i$, then there exists s' such that $(r,s') \in S_i(D_i)$,
- if $(r,s) \in S_i(D_i)$ and $(r,s') \in S_i(D_i')$, then $s \geq s'$.

It is straightforward to see that Assumption 4 implies Assumption 3. Therefore, under Assumptions 1, 2 and 4, the optimal mechanisms are donor monotonic. Moreover, in this case, if a patient reports a subset of her donors and still receives the same amount of blood, then the second condition in Assumption 4 implies that her donors do not donate less blood. Hence, we have the following result.

Theorem 3. Under Assumptions 1, 2, and 4, every optimal mechanism is strongly donor monotonic.

One important circumstance under which strong donor monotonicity can be achieved is when the feasible schedule correspondences feature exogenous exchange rates, in the sense that for every possible amount of blood received in a feasible schedule set, there is a unique amount of supply associated with it. That is, for every $i \in I$, $D_i \in \mathcal{D}_i$ and $(r,s) \in S_i(D_i)$, there does not exist $s' \neq s$ such that $(r,s') \in S_i(D_i)$. In this case, Assumption 3 and Assumption 4 are equivalent.

Remark 2. Suppose that the exchange rates are exogenous. Then Assumptions 1, 2, and 3 pin down a particular class of feasible schedule correspondences. Assume that for every $i \in I$, $D_i \neq \emptyset$ for some $D_i \in \mathcal{D}_i$, then Assumptions 1, 2, and 3 are satisfied if and only if the following is true for every $i \in I$:

- for every $D_i \in \mathcal{D}_i$ such that $S_i(D_i) \neq \{(0,0)\}$, there exist $\underline{s}_i(D_i), \overline{r}_i(D_i) \in \mathbb{Z}_+$, where $\underline{s}_i(D_i) \leq |D_i|$, $\underline{s}_i(D_i) = 0$ if $\underline{n}_i = 0$, and $\underline{n}_i \leq \overline{r}_i(D_i) \leq \overline{n}_i$, such that

$$S_i(D_i) = \{(r,s) \in W_i : s - \underline{s}_i(D_i) = r - \underline{n}_i;\ s \leq |D_i|;\ \text{and } \underline{n}_i \leq r \leq \overline{r}_i(D_i)\};$$

- for every $D_i \in \mathcal{D}_i$ and $D_i' \subseteq D_i$ such that $S_i(D_i) \neq \{(0,0)\}$ and $S_i(D_i') \neq \{(0,0)\}$, $\underline{s}_i(D_i) \geq \underline{s}_i(D_i')$ and $\overline{r}_i(D_i) \geq \overline{r}_i(D_i')$, and
- for every $D_i \in \mathcal{D}_i$ and $D_i' \subseteq D_i$, $S_i(D_i) = \{(0,0)\}$ implies $S_i(D_i') = \{(0,0)\}$.

Thus, if a patient i participates in the program, then she has to supply $\underline{s}_i(D_i)$ units to receive her minimum guarantee. Beyond this schedule, she has to supply one additional unit for each additional unit received, with the maximal amount received being restricted by $\overline{r}_i(D_i)$. We refer to such feasible schedule correspondences as *two-part tariffs*, which include both the one-for-one exchange rate policy and the Xi'an policy in Example 1 as special cases. We give another exampleartlo(,)-350Figu(,)-384(whExample)]TJ 0 0 1 rg 0 3

Figure 5: An illustration of the two-part tariff policy. The patient i has to supply two units to receive her minimum guarantee of $\underline{n}_i = 3$ units. The first four graphs illustrate $S_i(D_i)$ for $|D_i| \in \{0, \ldots, 4\}$, while the last graph shows how the feasible schedule set changes as the number of donors increases.

Strong donor monotonicity of optimal mechanisms can also be achieved under feasible schedule correspondences that incorporate endogenous exchange rates. An example is given in Figure 6.

Figure 6: An illustration of a feasible schedule correspondence satisfying Assumptions 1, 2, and 4. Exchange rates are endogenous when the patient has three or five donors.

4.3 Comparative Statics

In establishing the donor monotonicity of the optimal mechanisms, we need Assumption 3, which requires that if a patient i brings forward a donor set D_i larger than D_i', i.e., $D_i' \subseteq D_i$, then $S_i(D_i)$ is weakly more favorable than $S_i(D_i')$ at D_i'. A weakly more favorable feasible schedule set is given to the patient to incentivize her to report the full set of donors. It is then natural to consider the effect of making her feasible schedule set weakly more favorable, while keeping her donor set fixed. To this end, we introduce a notation to denote the possibility of changing the underlying feasible schedule correspondences. For a given profile of feasible schedule correspondences $S = (S_i)_{i \in I}$ and an optimal mechanism f , let f D j S be the outcome of f for any D 2 D under S.

Theorem 4.

and endogenous exchange rates, bringing rigor and transparency to the allocation system.

plemental Material shows that a priority mechanism may not be donor monotonic under such an exogenous exchange rate policy, due to L-convexity not being satis ed.

However, we can generate endogenous exchange rate policies that closely approximate the two-for-one exchange rate, such that under these policies the optimal mechanisms are feature of this problem in which patients arrive over time is less crucial in implementation once initial conditions are set.

We propose the establishment of a donor registry system that allows patients to register information about their potential replacement donors at the time they are seeking blood. A potential donor registered into the system may later be utilized depending on her blood type or the amount of blood the patient will end up receiving. When a certain threshold of potential donors is reached (for example, this could be daily for logistical reasons),[38] one of our practical optimal mechanisms is implemented to determine the actual blood assignment of non-urgent patients together with the potential replacement donors that are requested to donate blood. After the chosen donors donate and the blood is tested and processed, the medical procedures requiring transfusions will be conducted in the following days, or if the slack is large in the blood bank, then the replacement donor blood can be used to replenish the inventory after the patients receive blood in the preceding days.[39]

some amount of donor plasma and thus follow the commonly practiced ABO-identical and Rh D-compatible protocol. We consider three patient set sizes $|I| = 25, 50, 100$, representing medium to large hospital systems and their blood banks. Each patient i is assumed to need a maximum of f_i units, determined by an independent and identical draw from the uniform distribution with the support set f

Figure 10: Total units of blood transfused to the patients in the simulations for patient set sizes $|I| = 25, 50, 100$ as a function of (the ratio of the maximum units in the blood bank inventory to the maximum number of replacement donors), under the three allocation protocols.

panels) and the marginal distribution of net demand calculated as the difference between

Figure 11: Distributions of transfusion units (top panels) and net demand (bottom panels) in the simulations for $|I| = 50$, when $= 0$ (left panels) and $= \frac{1}{10}$ (right panels).

For $|I| = 50$ and $= 0$, only 33 units of blood are transfused (Figure 10), with more than 26 patients receiving no blood and no patient receiving more than 4 units (Figure 11 to the complementarities in dual organ exchanges in Ergin et al. (2017). However, the one-for-one exchange rate is not crucial in our model while it is important in the latter study. The di erences in institutional details between solid organ exchange applications and our main application are explained in Section 2. Our two donor monotonicity notions would reduce to the donor monotonicity notion introduced in Roth et al. (2005) if patients had unit demand and the exchange rate were one-for-one.

The WHO guidelines suggest that blood should only come from VNRDs and economic incentives can adversely a ect both blood safety and blood donation. The position of the WHO has been questioned based on recent evidence (Lacetera et al., 2013). In particular, Lacetera et al. (2012) provide evidence from a natural eld experiment showing that economic incentives have a positive e ect on voluntary donation and can encourage pro-social behavior. Additionally, Slonim et al. (2014) also study blood donation from an economic perspective, and discuss methods to increase blood supply and improve the supply and demand balance without market prices. Pay-it-forward and pay-it-backward incentive schemes for encouraging COVID-19 convalescent plasma donation have recently been proposed by Kominers et al. (2020) in a market design context.[43]

There are not many papers on mechanism or market design for multi-unit exchange of indivisible goods, even under the restriction of one-for-one exchange rate. Besides Ergin et al. (2017), two notable exceptions are Manjunath and Westkamp (2021), who study shift exchanges for medical doctors and other professionals as a market design problem,[44] and Andersson et al. (2021), who consider the design of time banks or favor barter markets to be cleared by centralized clearinghouses.[45] Our paper as well as Andersson et al.

[43] They propose issuing vouchers for the convalescent plasma donation of patients who recover from COVID-19 that can be used by these donors' family members who may become sick in the future to gain prioritized access to plasma therapy or for their own treatment, if they are still sick. Since one donor can donate plasma that can treat more than one patient, the system can collect enough plasma to treat all patients. Their paper inspects the steady-state analysis of a stylized large-market model, while ours is on mechanism design in a nite environment.

[44] In Manjunath and Westkamp (2021), for each agent there are three indi erence classes of objects: desirable objects, undesirable objects that she is endowed with, and undesirable objects that she is not endowed with. This trichotomous preference domain is more general than ours, and suits their application of shift exch454(tuuo39t)-(exc)2duTJ 0 -11.9526 Tf 8.441 -3.615 Td [(In)-3685.112 -22.2nia85.112 -2io39te

(2021) substantially generalizes the priority mechanism introduced for bilateral kidney compatibility-based preferences model.[50]

In closing, it is our hope that in addition to developing the theory for efficient blood allocation mechanisms with good incentive properties, our approach will be an important first step toward a blueprint of transparent, equitable, and systematic replacement donor programs that are in line with the goals of the WHO. Relaxing the constraints imposed by fixed exchange rates, this approach can help to overcome important practical frictions such as coercion and emerging black markets in places where these programs are not adequately organized.

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hidden endowments in addition to what they acquire by exchange. However, in our problem blood needs to be tested and processed before transfusion, and thus no patient can use her hidden donors' blood. Atlamaz and Klaus (2007) consider a multi-object assignment setting and show that individually rational and efficient rules are generally manipulable via hiding or destroying endowments. Sertel and Özkal-Sanver (2002) study manipulation via endowments in the two-sided matching market. In the context of airline landing slot assignment, Schummer and Abizada (2017) show that while any efficient rule is manipulable via slot destruction, a positive result emerges under a weaker form of efficiency suitable for that context.

[50] Besides its theory contributions, more broadly our paper is an addition to the field of market design in which economists have recently contributed to the design of market institutions, such as entry-level

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Online Appendix

A Proofs

A.1 Proof of Theorem 1

Theorem 1 follows from the fact that every sequential targeting mechanism is a weighted maximal mechanism, which is proved in Appendix C.1 in Supplemental Material.

A.2 Proof of Theorem 2

We first show that Assumptions 1, 2 and 3 imply the following two assumptions on the feasible schedule correspondences, which will be useful in our proof.

Assumption 1′. For every $i \in I$, $D_i \in \mathcal{D}_i$, and $(r;s);(r';s') \in S_i(D_i)$, we have

1. If $r' > r$ and $s' > s$, then

$$(r+1; s+1) \in S_i(D_i) \quad \text{and} \quad (r'-1; s'-1) \in S_i(D_i):$$

2. If $r' > r$ and $s' \leq s$, then

$$(r+1; s) \in S_i(D_i) \quad \text{and} \quad (r'-1; s') \in S_i(D_i):$$

3. If $s' > s$ and $r' \leq r$, then

$$(r; s+1) \in S_i(D_i) \quad \text{and} \quad (r'; s'-1) \in S_i(D_i):$$

Assumption 2′. For every $i \in I$, $D_i; D_i' \in \mathcal{D}_i$ with $D_i' \subseteq D_i$, $(r;s) \in S_i(D_i)$ and $(r';s') \in S_i(D_i')$, we have

1. If $r' > r; s' > 0$ and $s < D_i$, then

$$(r+1; s+1) \in S_i(D_i) \quad \text{and} \quad (r'-1; s'-1) \in S_i(D_i'):$$

2. If $r' > r$ and $s' \leq s$, then

$$(r+1; s) \in S_i(D_i) \quad \text{and} \quad (r'-1; s') \in S_i(D_i'):$$

Lemma 1. Assumption 1′ and Assumption 2′ are satisfied.

Proof of Lemma 1. Consider any $i \in I$ and $D_i \in \mathcal{D}_i$. Let $S_i(D_i) = S$. For any $x; y \in W_i$, where $x = (r;s)$ and $y = (r';s')$, denote $dis(x;y) = r' - r + s' - s$, and $y > x$ if $r' > r$ and $s' > s$. Suppose that $x = (r;s) \in S$, $y = (r';s') \in S$, and $y > x$. We want to first show that $(r+1; s+1) \in S$. If $dis(x;y) = 2$, then we are done. If $dis(x;y) > 2$, then consider $z = \frac{x+y}{2} > x$. By TJ/F50 11.9552 Tf 26.659 0 Td [(rnd [(p(a

$2 \le dis(x,z) < dis(x,y)$. If $dis(x,z) > 2$, we can repeat the argument and find $z' \in S$ such that $z' > x$ and $2 \le dis(x,z') < dis(x,z)$. Continuing in this fashion, in the end we must have $(r+1, s+1) \in S$. By symmetric arguments, it can be shown that $(r'-1, s'-1) \in S$. So Condition 1 in Assumption 1′ is satisfied.

Next we show Condition 2. Suppose that $x = (r,s) \in S$, $y = (r',s') \in S$, $r' > r$ and $s' \le s$. First, we argue that there exists $s'' \le s$ such that $(r+1, s'') \in S$. If $r' = r+1$, we are done. If $r' > r+1$, then consider $\frac{x+y}{2} = (r_1, s_1)$. We have $r' > r_1 > r$ and $s_1 \le s$. By Assumption 1, $(r_1, s_1) \in S$. If $r_1 > r+1$, we can repeat the argument and find $(r_2, s_2) \in S$ such that $r_1 > r_2 > r$ and $s_2 \le s$. Therefore, eventually we have $(r+1, s'') \in S$ for some $s'' \le s$. Denote $z = (r+1, s'')$. If $s'' < s$, consider $\frac{x+z}{2} = (r+1, s_3)$. Then $s'' < s_3 \le s$. By Assumption 1, $(r+1, s_3) \in S$. If $s_3 < s$, we can repeat the argument and find some s_4 such that $(r+1, s_4) \in S$ and $s_3 < s_4 \le s$. Therefore, we must have $(r+1, s) \in S$. By symmetric arguments, it can be shown that $(r'-1, s') \in S$. Finally, Condition 3 in Assumption 1′ can be shown in a similar way as the proof of Condition 2.

To show Assumption 2′, consider any $i \in I$, $D_i, D_i' \in D_i$ with $D_i' \subseteq D_i$, $(r,s) \in S_i(D_i)$ and $(r',s') \in S_i(D_i')$.

Suppose that $r' > r$, $s' > 0$ and $s < |D_i|$. Since $r' > 0$, by the definition of feasible schedule correspondences, $S_i(D_i') \neq \{(0,0)\}$ and $r' \ge \underline{n}_i$. Then by Assumption 3 (Non-diminishing favorability in donors), there exists s_1 such that $(r', s_1) \in S_i(D_i)$. Since $r' > r$ and $s < |D_i|$, by Assumption 2 (Feasibility of positive price), there exists $s_2 > s$ such that $(r', s_2) \in S_i(D_i)$. Then, given that $(r', s_2) > (r,s)$, it follows from Condition 1 in Assumption 1′ that $(r+1, s+1) \in S_i(D_i)$. This also implies that $S_i(D_i) \neq \{(0,0)\}$, and hence $r \ge \underline{n}_i$. Recall that $S_i(D_i') \neq \{(0,0)\}$. So there exists s_3 such that $(\underline{n}_i, s_3) \in S_i(D_i')$. Since $r' > r \ge \underline{n}_i$ and $s' > 0$, by Assumption 2, there exists $s_4 < s'$ such that $(\underline{n}_i, s_4) \in S_i(D_i')$. Then, given that $(r', s') > (\underline{n}_i, s_4)$, it follows from Condition 1 in Assumption 1′ that $(r'-1, s'-1) \in S_i(D_i')$.

It remains to show Condition 2 in Assumption 2′. Suppose that $r' > r$ and $s' \le s$. Then $r' \ge \underline{n}_i$. By Assumption 3, there exists $s_1 \le s' \le s$ such that $(r', s_1) \in S_i(D_i)$. It follows from Condition 2 in Assumption 1′ that $(r+1, s) \in S_i(D_i)$. Then, we argue that $(r, s') \in S_i(D_i)$. This is true if $s' = s$. Suppose that s'

$(r; s_3) \in S_i(D_i')$. Since $(r; s') \in S_i(D_i)$ and $s' \leq |D_i'|$, by Assumption 3, there exists $s_4 \geq s'$ such that $(r; s_4) \in S_i(D_i')$. As $(r', s') \in S_i(D_i')$, $r' > r$ and $s' \leq s_4$, it follows from Condition 2 in Assumption 1' that $(r' - 1; s') \in S_i(D_i')$. ■

We introduce new machinery to prove this theorem. In particular, we will construct *extended problems* in which each blood type has a replica and there are some new dummy agents. Such a construction mainly serves two purposes: it helps us represent allocations as *matchings*, which specify the donors that each patient receives blood from; it allows us to focus on the simple case where no patient receives blood from her own (compatible) donors.

First, we treat the blood bank b as if it were a *pseudo patient* and introduce a donor set for it. It has a set of (volunteer non-remunerated) donors D_b, where for each blood type $X \in B$ the blood bank has denoted as M_i by a slight abuse of notation, such that

1. $M_i \cap M_j = \emptyset$ for every $i; j \in \hat{I}$ with $i \neq j$, and $\cup_{i \in \hat{I}} M_i = \hat{D}$,
2. for every $i \in \hat{I} \setminus \{b\}$ and $d \in M_i \setminus D_i$, $_d \in \hat{C}(_i)$,
3. for every $i \in \hat{I} \setminus \{b\}$, ($M_i \setminus D_i$; 2 3.431 Td [(M)]T (or)-327(ev)27(ery)]TJ/F50 11.9552

anisms. For each $\hat{X} \in B$, let $W_{i_{\hat{X}}} = \{0, 1, \ldots, \bar{n}_{i_{\hat{X}}}\}^2$. A vector ^

1. $_d = X$ for every $d \in M_{i_{\hat{X}}}^{X}$,
2. $(d) = 1$ for every $d \in M_{i_{\hat{X}}}^{X} \setminus D_b$, and
3. $M_{i_{\hat{X}}}^{X} = \sum_{j \in I : X \in C(\ _j)} M_j^{\hat{X}}$.

Then we construct a matching M for $\hat{D}$ as follows:

- for each $j \in I$, $M_j = \bigcup_{X \in C(\ _j)} M_j^{\hat{X}} \cup \{d \in D_j : (d) = 0\}$,
- for each $X \in B$, $M_{i_{\hat{X}}} = M_{i_{\hat{X}}}^{X} \cup \left(D_{i_{\hat{X}}} \setminus \bigcup_{j \in I : X \in C(\ _j)} M_j^{\hat{X}} \right)$, and
- $M_b = \hat{D} \setminus \left((\bigcup_{j \in I} M_j) \cup (\bigcup_{X \in B} M_{i_{\hat{X}}}) \right)$.

Therefore, each patient $j \in I$ is matched with $_X(j)$ dummy donors of type $\hat{X}$ for every $X \in C(\ _j)$ (recall that for the extended problem, $\hat{X} \in \hat{C}(\ _j)$), and j's own donor d is matched with j if and only if $(d) = 0$. Moreover, for each dummy patient $i_{\hat{X}}$, the number of X donors from $I \cup \{b\}$ matched with her is equal to the number of her $\hat{X}$ donors that are not matched with her (recall that $\hat{C}(\hat{X}) = \{X\}$). Hence, M is a well-defined matching for $\hat{D}$ and it is welfare equivalent to .

<u>Part 2.</u> On the other hand, let $M \in \mathcal{M}(\hat{D})$. Construct as follows:

- for each $j \in I$ and $X \in C(\ _j)$, let $_X(j) = \left| d \in M_j \setminus D_j : \ _d \in \{ \right.$

$D \in \mathcal{D}$, $f(D)$ and $F(\hat{D})$ are welfare equivalent.

Let $D \in \mathcal{D}$. By the claim above, there exists $\ \in A(D)$ that is welfare equivalent to $F(\hat{D})$. By the definition of f, $w(f(D)) \quad w(\ )$, where $w(\ ) = \hat{}$

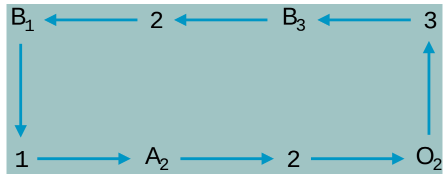

Figure 12: Suppose that $I = \{1, 2, 3\}$, with $\tau_1 = A$, $\tau_2 = B$ and $\tau_3 = O$, $\hat{D} = \hat{D}'$, and the donor sets are given by $D_1 = \{B_1\}$, $D_2 = \{A_2, O_2\}$, $D_3 = \{B_3\}$ and $D_b = \emptyset$, where a type-X donor of a patient i is denoted as X_i. For simplicity, we omit the dummy patients. For every $i \in I$, $\overline{n}_i = 1$, $\underline{n}_i = 0$ and the exchange rate is one-for-one. Assume ABO-identical transfusion. Consider the following two matchings M and M': $M_1 = \{B_1\}$, $M_2 = \{A_2, B_3\}$, $M_3 = \{O_2\}$ and $M_b = \emptyset$; $M'_1 = \{A_2\}$, $M'_2 = \{O_2, B_1\}$, $M'_3 = \{B_3\}$ and $M'_b = \emptyset$. The above graph gives a cycle C from M to M', and we have $M + C = M'$ and $M' - C = M$.

remove d_{t-1} from M_{i_t}. Condition 1 above guarantees that this leads to a well-defined function, which we denote as $M + C$ and satisfies Conditions 1 and 2 in the definition of a matching (for $\hat{D}$). The patients involved in the cycle may not be distinct. But Condition 4 above says that if a patient $i \in \hat{I} \setminus \{b\}$ appears twice in the cycle, then her schedule is not affected by the exchanges, i.e., the amount of blood received and the amount of blood supplied remain the same. Note that this condition also implies that any patient cannot appear more than twice in the cycle. Finally, if a patient $\in \hat{I} \setminus \{b\}$ is assigned a different schedule under $M + C$ than under M, then she appears only once in the cycle, and she either receives one more unit and supplies one more unit, or receives one less unit and supplies one less unit. Then Conditions 2 and 3 above imply Condition 3 in the definition of a matching. Therefore $M + C$ is a matching for $\hat{D}$. Similarly, we could instead start from M' and assign each patient in the cycle the donor she is pointed by (who is one of her M matches) instead of the donor she points to (who is one of her M' matches). That is, for each $t \in \{1, \ldots, t\}$, add d_{t-1} to M'_{i_t} and remove d_t from M'_{i_t}. These exchanges also lead to a well-defined matching for $\hat{D}'$, denoted as $M' - C$. In Figure 12, we give an example of a cycle and the construction of new matchings using this cycle.

It is wise to note that the cycle operations do not necessarily make all patients involved better off or worse off. Instead, they generate new matchings that are closer to each other in terms of the matches of the patients.

Another concept similar to a cycle is a chain. A chain from M to M' is a directed graph of patients and donors in which each patient/donor points to the next donor/patient in the chain, and is represented as a list $C = (i_1, d_1, \ldots, i_{t-1}, d_{t-1}, i_t)$, $t \geq 2$, such that

1. For every $t \in \{1, \ldots, t\}$, $i_t \in \hat{I}$ such that if $i_t = b$ then $t \in \{1, t\}$, and $i_1 \neq i_t$.

2. For every $t \in \{1, \dots, t-1\}$, $d_t \in M^0_{i_t} \setminus M_{i_t}$ and $d_t \in M_{i_{t+1}}$.
3. For every $t \in \{2, \dots, t-1\}$, if $d_{t-1} \in D_{i_t}$ and $d_t \notin D_{i_t}$, then

   $(|M_{i_t} \setminus D_{i_t}| + 1;\ |D_{i_t} \setminus M_{i_t}| + 1) \in S_{i_t}(D_{i_t})$ and $(|M^0_{i_t} \setminus D^0_{i_t}| - 1;\ |D^0_{i_t} \setminus M^0_{i_t}| - 1) \in S_{i_t}(D^0_{i_t})$:
4. For every $t \in \{2, \dots, t-1\}$, if $d_{t-1} \notin D_{i_t}$, and $d_t \in D_{i_t}$, then

   $(|M_{i_t} \setminus D_{i_t}| - 1;\ |D_{i_t} \setminus M_{i_t}| - 1) \in S_{i_t}(D_{i_t})$ and $(|M^0_{i_t} \setminus D^0_{i_t}| + 1;\ |D^0_{i_t} \setminus M^0_{i_t}| + 1) \in S_{i_t}(D^0_{i_t})$:
5. If $i_t \neq b$, then

   $(|M_{i_t} \setminus D_{i_t}|;\ |D_{i_t} \setminus M_{i_t}| + 1) \in S_{i_t}(D_{i_t})$ and $(|M^0_{i_t} \setminus D^0_{i_t}|;\ |D^0_{i_t} \setminus M^0_{i_t}| - 1) \in S_{i_t}(D^0_{i_t})$

   when $d_{t-1} \in D_{i_t}$, and

   $(|M_{i_t} \setminus D_{i_t}| - 1;\ |D_{i_t} \setminus M_{i_t}|) \in S_{i_t}(D_{i_t})$ and $(|M^0_{i_t} \setminus D^0_{i_t}| + 1;\ |D^0_{i_t} \setminus M^0_{i_t}|) \in S_{i_t}(D^0_{i_t})$

   when $d_{t-1} \notin D_{i_t}$.
6. If $i_1 \neq b$, then

   $(|M_{i_1} \setminus D_{i_1}|;\ |D_{i_1} \setminus M_{i_1}| - 1) \in S_{i_1}(D_{i_1})$ and $(|M^0_{i_1} \setminus D^0_{i_1}|;\ |D^0_{i_1} \setminus M^0_{i_1}| + 1) \in S_{i_1}(D^0_{i_1})$

   when $d_1 \in D_{i_1}$, and

   $(|M_{i_1} \setminus D_{i_1}| + 1;\ |D_{i_1} \setminus M_{i_1}|) \in S_{i_1}(D_{i_1})$ and $(|M^0_{i_1} \setminus D^0_{i_1}| - 1;\ |D^0_{i_1} \setminus M^0_{i_1}|) \in S_{i_1}(D^0_{i_1})$

   when $d_1 \notin D_{i_1}$.
7. If $i_t = i_{t^0} = i$ for some $t; t^0$ such that $1 < t < t^0 < t$, then either (i) $d_t; d_{t-1} \in D_i$ and $d_{t^0}; d_{t^0-1} \notin D_i$, or (ii) $d_t; d_{t-1} \notin D_i$ and $d_{t^0}; d_{t^0-1} \in D_i$.

   If i_t^{+1}

Figure 13: Suppose that $I = f1;2;3g$ with $\;_1 = \;_2 = A$ and $\;_3 = B$. The donor sets in two extended problems $\hat{D}$ and $\hat{D}^0$ are given by $D_1 = fB_1g$, $D_1^0 = ;$, $D_2 = D_2^0 = ;$, $D_3 = D_3^0 = fA_3g$ and $D_b = fA_b; A_b^0; B_bg$, where X_i (or X_i^0) denotes a type-X donor of patient i. For simplicity, we omit the dummy patients. For every i 2 I, $\overline{n}_i = 2$, $\underline{n}_i = 0$ and the feasible schedules are

Patients:	1 (A)	2 (A)	3 (B)	4 (O)	5 (AB)	6 (A)	7 (O)	b
Donors:	B_1 B_1' AB_1 $\boxed{O_1}$	B_2	A_3	A_4	A_5 O_5	AB_6	A_7	A_b A_b' O_b
$(\underline{n}_i; \overline{n}_i)$:	$(0; 3)$	$(1; 3)$	$(0; 3)$	$(0; 3)$	$(0; 3)$	$(0; 3)$	$(0; 3)$	
M	B_1 AB_1 A_7 A_b'	A_b	A_3	A_4 O_b	A_5 O_5	AB_6	O_1	B_1' B_2
M'	B_1' A_7 A_b' A_b	B_2 A_3 A_4	B_1	O_b	AB_1 AB_6	A_5	O_5	$\emptyset$
M''	B_1 O_1 A_7 A_b'	A_b	A_3	A_4 O_b	A_5 AB_1	AB_6	O_5	B_1' B_2

Table 2: The patients, their donors, the minimum guarantees and the maximum needs for Example 2. When Patient 1 truthfully reports his donor set, the matching M is obtained. When he conceals his donor O_1, the matching M' is obtained, in which he receives more blood. M'' is another matching that we explain in the example.

necessary condition for any rule that is not donor monotonic. Using this result, we show every optimal rule is donor monotonic (Lemma 4), which concludes the proof.

Lemma 3. Consider any $D, D' \in \mathcal{D}$ and $i \in I$ such that $D_i' \subseteq D_i$, $|D_i \setminus D_i'| = 1$, and $D_j' = D_j$ for every $j \in I \setminus \{i\}$. If $M \in \mathcal{M}(\hat{D})$, $M' \in \mathcal{M}(\hat{D}')$, and $|M_i' \setminus D_i'| > |M_i \setminus D_i|$, then there exists a cycle or a chain from M to M'.

The proof of this lemma is rather involved. We illustrate the ideas behind the proof using an example first. The example only demonstrates substantially different cases in the construction of a cycle or a chain in the proof, as some of the considered cases use similar constructions.

Example 2. Suppose that $I = \{1, \ldots, 7\}$. We omit the dummy patients for simplicity. The first row in Table 2 gives the blood type of each real patient $i \in I$. The second row gives the donor set D_i for each $i \in I \cup \{b\}$, where X_i (or X_i') denotes a type-X donor of patient i. Let $\overline{n}_i = 3$ for every $i \in I$, $\underline{n}_2 = 1$ and $\underline{n}_i = 0$ for every $i \in I \setminus \{2\}$. Assume ABO-identical transfusion.

We will also consider the situation in which Patient 1 conceals his donor O_1.[57] Let

$$D_1' = D_1 \setminus \{O_1\},$$

and $D_i' = D_i$ for every $i \in I \setminus \{1\}$. Finally, for every $i \in I$ and every $D_i'' \subseteq D_i$, let

$$S_i(D_i'') = \{(r, s) : \underline{n}_i \leq r \leq \overline{n}_i,\ 0 \leq s \leq |D_i''|,\ s \leq r\}.$$

The last three rows in Table 2 specify three matchings M, M' and M'', where M and M'' are matchings for $\hat{D}$ and M' is a matching for $\hat{D}'$. Given that Patient 1 receives more blood under M' than under M, we discuss how to find a cycle or a chain from M to M' using an iterative "pointing procedure from M to M'" that is formally defined in the proof of Lemma 3. At each step of the procedure, a patient points to a donor that he

[57] Assume that the patients are male and the donors are female in this example.

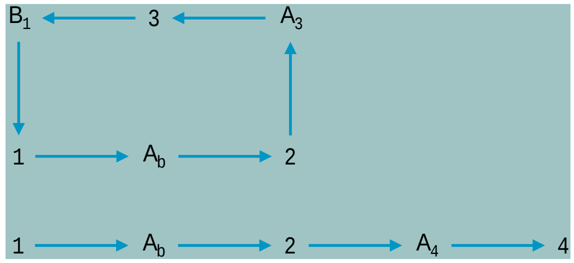

Figure 14: A cycle and a chain from M to M^0 found using the pointing procedure from M to M^0 (illustrating Case 2 and Case 3 in the proof of Lemma 3, respectively).

Recall that Patient 2 could also point to A_4. If Patient 2 points to A_4, then A_4 points to Patient 4. Given that Patient 4 cannot point to his own donor and he does not receive more blood under M^0, we stop here. In this case, a chain is identified as in the graph in Figure 14. This construction corresponds to Case 3 in the proof of Lemma 3. This c7

Figure 15: A cycle from M to M^0 and another directed graph, a pseudo-cycle from M to M^0, in the modi ed example. Both are constructed by reversing the edge orientation of the graphs found using the pointing procedure from M^0 to M (illustrating Subcase 4.1 and Subcase 4.5 in the proof of Lemma 3, respectively).

15. This construction corresponds to Subcase 4.1 in the proof of Lemma 3.

On the other hand, if Patient 5 points to O_5, then O_5 points to Patient 7, who

1. If $d_{t-1} \in D_{i_t}$: We have two cases:
   (a) If there exists $d \in D_{i_t}$ such that $d \in M'_{i_t} \setminus M_{i_t}$: Then at Step t, let i_t point to $d_t = d$, and d_t point to i_{t+1} such that $d_t \in M_{i_{t+1}}$.[58]
   (b) If there does not exist $d \in D_{i_t}$ such that $d \in M'_{i_t} \setminus M_{i_t}$: Then $|D'_{i_t} \setminus M'_{i_t}| > |D_{i_t} \setminus M_{i_t}|$. We have two subcases:
      i. If $|M'_{i_t} \setminus D'_{i_t}| > |M_{i_t} \setminus D_{i_t}|$: Then there exists $d_t \notin D_{i_t}$ such that $d_t \in M'_{i_t} \setminus M_{i_t}$. At Step t, let i_t point to d_t, and d_t point to i_{t+1} such that $d_t \in M_{i_{t+1}}$.
      ii. If $|M'_{i_t} \setminus D'_{i_t}| \leq |M_{i_t} \setminus D_{i_t}|$: Then i_t does not point and stop at i_t at Step $t-1$.
2. If $d_{t-1} \notin D_{i_t}$: We have two cases:
   (a) If there exists $d \notin D_{i_t}$ such that $d \in M'_{i_t} \setminus M_{i_t}$: Then at Step t, let i_t point to $d_t = d$, and d_t point to i_{t+1} such that $d_t \in M_{i_{t+1}}$.
   (b) If there does not exist $d \notin D_{i_t}$ such that $d \in M'_{i_t} \setminus M_{i_t}$: Then $|M'_{i_t} \setminus D'_{i_t}| < |M_{i_t} \setminus D_{i_t}|$. We have two subcases:
      i. If $|D'_{i_t} \setminus M'_{i_t}| < |D_{i_t} \setminus M_{i_t}|$: Then there exists $d_t \in$

The first circumstance implies that any patient can be pointed at most three times in the procedure. Hence, the procedure always stops in a finite number of steps.

We consider the following four cases based on these circumstances. Case 1 and Case 2 cover the first two circumstances in order and show the existence of a cycle in each case. Case 3 covers the third and the fourth circumstances together when i_1 does not supply more blood under M' than under M, and shows the existence of a chain. Finally, Case 4 covers the third and the fourth circumstances together when i_1 supplies more blood under M' than under M, and shows the existence of a cycle or a chain. This is the most involved case and we will handle it the last.

Case 1. The procedure stops at d_t at Step t.

Then for some $\underline{t} < t$, $i_{\underline{t}} = i_t \notin \{i_1, b\}$ and neither of the following is true:

1. $d_{\underline{t}}, d_{\underline{t}-1} \in D_{i_{\underline{t}}}$ and $d_t, d_{t-1} \notin D_{i_{\underline{t}}}$.
2. $d_{\underline{t}}, d_{\underline{t}-1} \notin D_{i_{\underline{t}}}$ and $d_t, d_{t-1} \in D_{i_{\underline{t}}}$.

We show that $(i_{\underline{t}}, d_{\underline{t}}, \ldots, i_{t-1}, d_{t-1})$ is a cycle from M to M'.

First, for any t' such that $\underline{t} < t' \leq t-1$, $i_{t'} \notin \{i_1, b\}$, since otherwise the procedure stops at $i_{t'}$ at Step $t'-1$. It follows that $D_{i_{t'}} = D'_{i_{t'}}$ for every t' such that $\underline{t} \leq t' \leq t-1$. By the construction of the pointing procedure from M to M', Condition 1 in the definition of a cycle is satisfied. Next, we show Condition 2 and Condition 3.

First, consider any t' such that $\underline{t} < t' \leq t-1$. If $d_{t'-1} \in D_{i_{t'}}$ and $d_{t'} \notin D_{i_{t'}}$, then by the construction, we have $|M'_{i_{t'}} \setminus D'_{i_{t'}}| > |M_{i_{t'}} \setminus D_{i_{t'}}|$ and $|D'_{i_{t'}} \setminus M'_{i_{t'}}| > |D_{i_{t'}} \setminus M_{i_{t'}}|$. Since

$(|M_{i_{t'}} \setminus D_{i_{t'}}|, |D_{i_{t'}} \setminus M_{i_{t'}}|) \in S_{i_{t'}}(D_{i_{t'}})$ and $(|M'_{i_{t'}} \setminus D'_{i_{t'}}|, |D'_{i_{t'}} \setminus M'_{i_{t'}}|) \in S_{i_{t'}}(D'_{i_{t'}}) = S_{i_{t'}}(D_{i_{t'}})$;

it follows from Assumption 1' that

$$(|M_{i_{t'}} \setminus D_{i_{t'}}|+1, |D_{i_{t'}} \setminus M_{i_{t'}}|+1) \in S_{i_{t'}}(D_{i_{t'}}) \text{ and } (|M'_{i_{t'}} \setminus D'_{i_{t'}}|-1, |D'_{i_{t'}} \setminus M'_{i_{t'}}|-1) \in S_{i_{t'}}(D'_{i_{t'}}).$$

Similarly, if $d_{t'-1} \notin D_{i_{t'}}$ and $d_{t'} \in D_{i_{t'}}$, then by the construction we have $|M'_{i_{t'}} \setminus D'_{i_{t'}}| < |M_{i_{t'}} \setminus D_{i_{t'}}|$ and $|D'_{i_{t'}} \setminus M'_{i_{t'}}| < |D_{i_{t'}} \setminus M_{i_{t'}}|$. It follows from Assumption 1' that

$$(|M_{i_{t'}} \setminus D_{i_{t'}}|-1, |D_{i_{t'}} \setminus M_{i_{t'}}|-1) \in S_{i_{t'}}(D_{i_{t'}}) \text{ and } (|M'_{i_{t'}} \setminus D'_{i_{t'}}|+1, |D'_{i_{t'}} \setminus M'_{i_{t'}}|+1) \in S_{i_{t'}}(D'_{i_{t'}}).$$

Second, consider $i_{\underline{t}}$. Suppose that $d_{t-1} \in D_{i_{\underline{t}}}$ and $d_{\underline{t}} \notin D_{i_{\underline{t}}}$. Then either $d_{\underline{t}-1} \in D_{i_{\underline{t}}}$ or $d_t \notin D_{i_{\underline{t}}}$, as the procedure stops at the donor d_t. Since we have either (i) $d_{t-1} \in D_{i_{\underline{t}}}$ and $d_t \notin D_{i_{\underline{t}}}$, or (ii) $d_{\underline{t}-1} \in D_{i_{\underline{t}}}$ and $d_{\underline{t}} \notin D_{i_{\underline{t}}}$, by the construction,

$$|M'_{i_{\underline{t}}} \setminus D'_{i_{\underline{t}}}| > |M_{i_{\underline{t}}} \setminus D_{i_{\underline{t}}}| \text{ and } |D'_{i_{\underline{t}}} \setminus M'_{i_{\underline{t}}}| > |D_{i_{\underline{t}}} \setminus M_{i_{\underline{t}}}|.$$

Then by Assumption 1',

$$(|M_{i_{\underline{t}}} \setminus D_{i_{\underline{t}}}|+1, |D_{i_{\underline{t}}} \setminus M_{i_{\underline{t}}}|+1) \in S_{i_{\underline{t}}}(D_{i_{\underline{t}}}) \text{ and } (|M'_{i_{\underline{t}}} \setminus D'_{i_{\underline{t}}}|-1, |D'_{i_{\underline{t}}} \setminus M'_{i_{\underline{t}}}|-1) \in S_{i_{\underline{t}}}(D'_{i_{\underline{t}}}).$$

That is, Condition 2 in the definition of a cycle is satisfied for $i_{\underline{t}}$. By similar arguments, it can be shown that Condition 3 is also satisfied for $i_{\underline{t}}$.

It remains to show Condition 4. If $i_t = i_{t'}$ and $\underline{t} < t < t' \leq \bar{t} - 1$, then either (i) $d_t, d_{t-1} \in D_{i_t}$ and $d_{t'}, d_{t'-1} \notin D_{i_t}$, or (ii) $d_t, d_{t-1} \notin D_{i_t}$ and $d_{t'}, d_{t'-1} \in D_{i_t}$, since otherwise

$2'$. Finally, we verify Condition 7 for i_1 and i_t. For any $t' \in \{2, \ldots, t-1\}$, $i_1 \neq i_{t'}$, since otherwise the procedure stops at an earlier step. Suppose that $i_{t'} = i_t$ for some $t' \in \{2, \ldots, t-1\}$. Then $i_{t'} = i_t \neq b$. First consider the case that $d_{t'-1} \in D_{i_t}$. If $d_{t'} \in D_{i_t}$, then, given that $d_{t'} \in M'_{i_t} \setminus M_{i_t}$, $i_{t'} = i_t$ should point to this donor (or some other donor of her own) at Step t', which contradicts to the fact that the pointing procedure stops at i_t. So $d_{t'} \notin D_{i_t}$. Then $d_{t'-1} \notin D_{i_t}$, since otherwise $i_{t'} = i_t$ should point to $d_{t'}$ (or some other donor that is not her own) at Step t'. In the case that $d_{t'-1} \notin D_{i_t}$, it can be similarly shown that $d_{t'}, d_{t'-1} \in D_{i_t}$. These are the crucial conditions to check; the other conditions can be shown similarly as in Case 1.

<u>Case 4</u>. The procedure stops at i_t at Step $t-1$, $i_t \neq i_1$, and $D'_{i_1} \setminus M'_{i_1} > D_{i_1} \setminus M_{i_1}$.

In this case, we may not have $(M_{i_1} \setminus D_{i_1} + 1;\ D_{i_1} \setminus M_{i_1}) \in S_{i_1}(D_{i_1})$, and hence $(i_1; d_1; \ldots; d_{t-1}; i_t)$ may not be a chain from M to M'.

Let $j_1 = i_1$. Since $D'_{j_1} \setminus M'_{j_1} > D_{j_1} \setminus M_{j_1}$, there exists a donor $c_1 \in D'_{j_1}$ such that $c_1 \in M_{j_1} \setminus M'_{j_1}$

and the following is not true: she is pointed by and points to her own donors in one instance, and is pointed by and points to donors who are not her own in the other instance,

- when some $j \notin \{j_1; b\}$ has appeared before in the pointing procedure from M to M', and in this previous appearance she is not i_t. Moreover, the following is not true: she is pointed by and points to her own donors in one instance, and is pointed by and points to donors who are not her own in the other instance,[60]
- when b is pointed,
- when some $j \notin \{j_1; b\}$ does not point,
- when j_1 is pointed.

Due to the first circumstance, the pointing procedure from M' to M also stops in a finite number of steps. Since we are seeking a cycle or a chain from M to M', after the procedure stops we reverse the orientation of the constructed edges in $(j_1; c_1; j_2; c_2; \ldots)$.

We consider the following five subcases based on these circumstances. Subcase 4.1

edges in the second graph should be reversed. Then $(j_t; c_{t-1}; \ldots; c_1; i_1; d_1; \ldots; i_{t-1}; d_{t-1})$ is a cycle from M to M^0.

<u>Subcase 4.3</u>: The procedure stops at j_t at Step $t-1$, and $j_t \neq j_1$.

Then either $j_t = b$ or the patient j_t does not point.

If $j_t = i_t = b$, then $(j_t; c_{t-1}; \ldots; c_1; i_1; d_1; \ldots; i_{t-1}; d_{t-1})$ is a cycle from M to M^0.

If it is not true that $j_t = i_t = b$, then $(j_t; c_{t-1}; \ldots; c_1; i_1; d_1; \ldots; d_{t-1}; i_t)$ is a chain from M to M^0. To see this, we verify $j_t \neq i_t$ and Condition 6 in the definition of a chain. First, assume to the contrary, $j_t = i_t$. Then $j_t = i_t \in \hat{I} \setminus \{j_1; b\}$. If $d_{t-1} \in D_{i_t}$, then $c_{t-1} \notin D_{i_t}$, since otherwise in the pointing procedure from M^0 to M, j_t should point to d_{t-1} (or some other donor of her own) at Step t

$d \notin D_{j_t}$ with $d \in M'_{j_t} \setminus M''_{j_t}$, and the pointing procedure from M' to M'' starts with j_t pointing to some $c \in D'_{j_t}$ with $c \in M''_{j_t} \setminus M'_{j_t}$. Since $c_{t-1} \notin M'_i$ for any $i \in \hat{I}$, the concealed donor c_{t-1} is not pointed in the pointing procedure from M'' to M'. Moreover, $c_{t-1} \in M''_{j_t}$ implies that c_{t-1}

construction of the chain C from M'' to M', we have $M'_{\ell_w} \setminus D'_{\ell_w} \supseteq M''_{\ell_w} \setminus D''_{\ell_w}$ and $D'_{\ell_w} \setminus M'_{\ell_w} \supseteq D''_{\ell_w} \setminus M''_{\ell_w}$. Then by Observation 2, $M'_{\ell_w} \setminus D'_{\ell_w} \supseteq M_{\ell_w} \setminus D_{\ell_w}$ and $D'_{\ell_w} \setminus M'_{\ell_w} \supseteq D_{\ell_w} \setminus M_{\ell_w}$. Hence it follows from Assumption 1′ that Condition 3 is satisfied. Condition 4 can be shown in a similar manner.

Next, consider Condition 5. Suppose that $\ell_w \neq b$ and $a_{w-1} \in D_{\ell_w}$. For simplicity, denote

- $(M_{\ell_w} \setminus D_{\ell_w};\ D_{\ell_w} \setminus M_{\ell_w}) = (r; s)$,
- $(M''_{\ell_w} \setminus D_{\ell_w};\ D_{\ell_w} \setminus M''_{\ell_w}) = (r''; s'')$, and
- $(M'_{\ell_w} \setminus D'_{\ell_w};\ D'_{\ell_w} \setminus M'_{\ell_w}) = (r'; s')$.

Condition 5 is clearly satisfied if $(r; s) = (r''; s'')$. Suppose that $(r; s) \neq (r''; s'')$. Then $\ell_w \neq j_t$. By the construction of the chain C, and

$$\hat{w}'' + \hat{w}(F(\hat{D})) - \hat{w}' \wedge \hat{w}'' + \hat{w}(F(\hat{D}) + C) - \hat{w}' ;$$

where $\hat{w}''$ is defined such that for each $k \in \{1, \dots, 2(|\hat{I}| - 1)\}$, $\hat{w}''_k = \min\{\hat{w}_k(F(\hat{D}') - C), \hat{w}_k(F(\hat{D}'))\}$. By Observation 1,

$$\hat{w}(F(\hat{D})) - \hat{w}' = \hat{w}(F(\hat{D}') - C) - \hat{w}''; \text{ and } \hat{w}(F(\hat{D}) + C) - \hat{w}' = \hat{w}(F(\hat{D}')) - \hat{w}''.$$

Therefore,

$$\hat{w}(F(\hat{D}') - C) \wedge \hat{w}(F(\hat{D}'));$$

contradicting to the definition of F. Hence, $F(\hat{D})$ and $F(\hat{D}) + C$ are welfare equivalent. Then by Lemma 3 again, there is a cycle or a chain C' from $F(\hat{D}) + C$ to $F(\hat{D}')$. By similar arguments as before, it can be shown that $(F(\hat{D}) + C) + C'$ and $F(\hat{D}) + C$ are welfare equivalent. Then $(F(\hat{D}) + C) + C'$ and $F(\hat{D})$ are welfare equivalent. This process can be continued infinitely, which leads to a contradiction since each additional cycle or chain addition generates a matching that is closer to $F(\hat{D}')$. ■

A.3 Proof of Theorem 4

We first show that, given any optimal mechanism, if a patient's feasible schedule set becomes weakly more favorable, then she cannot receive less blood. The proof of this part uses the same techniques as those in the proof of Theorem 2. We explain how to modify the previous arguments to prove it. First, we present the following condition regarding different feasible schedule correspondences, which is a counterpart of Assumption 2'.

Assumption 2''. Consider any two profiles of feasible schedule correspondences S and S'. For every $i \in I$ and $D_i \in \mathcal{D}_i$, if $S_i(D_i)$ is weakly more favorable than $S'_i(D_i)$ at D_i, then for any $(r; s) \in S_i(D_i)$ and any $(r'; s') \in S'_i(D_i)$, we have

1. If $r' > r; s' > 0$ and $s < D_i$, then
$$(r + 1; s + 1) \in S_i(D_i) \quad \text{and} \quad (r' - 1; s' - 1) \in S'_i(D_i):$$
2. If $r' > r$ and $s' \leq s$, then
$$(r + 1; s) \in S_i(D_i) \quad \text{and} \quad (r' - 1; s') \in S'_i(D_i):$$

Using arguments similar to those in the proof of Lemma 1, it can be shown that when Assumptions 1 and 2 are satisfied for all feasible schedule correspondences, Assumption 2'' is satisfied.

Second, we use the same construction of extended problems as before. For a given profile of feasible schedule correspondences $S = (S_i)$, $\geq^{r;s}$

denote the outcome matching of F

Lemma 6. Consider any $D \in \mathcal{D}$ and any patient $i \in I$. Suppose that S and S' are two profiles of feasible schedule correspondences such that $S_j(D_j) = S'_j(D_j)$ for all $j \in I \setminus \{i\}$, and $S_i(D_i)$ is weakly more favorable than $S'_i(D_i)$ at D_i. If M is a matching for $\hat{D}$ under S, M' is a matching for $\hat{D}$ under S', and $|M'_i \setminus D_i| > |M_i \setminus D_i|$, then there is a cycle or a chain from M to M'.

Using Assumptions 1′ and 2″, Lemma 6 can be proved in the same way as Lemma 3. Since there is no concealed donor, Case 4.5 in the proof of Lemma 3 cannot happen.

By arguments similar to those in the proof of Lemma 4, Lemma 5 can be proved using Lemma 6. Specifically, we prove by contradiction. Assume that there exist some optimal rule F, $D \in \mathcal{D}$, $i \in I$, S and S', such that $S_j(D_j) = S'_j(D_j)$ for all $j \in I \setminus \{i\}$, $S_i(D_i)$ is weakly more favorable than $S'_i(D_i)$ at D_i, and

$$|F_i(\hat{D}|S) \setminus D_i| < |F_i(\hat{D}|S') \setminus D_i| :$$

Then by Lemma 6, there is a cycle or a chain C from $F(\hat{D}|S)$ to $F(\hat{D}|S')$. It can be shown that $F(\hat{D}|S) + C$ is welfare equivalent to $F(\hat{D}|S)$. By Lemma 6 again, there is a cycle or a chain C' from $F(\hat{D}|S) + C$ to $F(\hat{D}|S')$. Then $(F(\hat{D}|S) + C) + C'$ is welfare equivalent to $F(\hat{D}|S)$

Supplemental Material

B The General Multi-unit Exchange Model under Private Information

The main theoretical results in the paper are independent of the blood allocation and transfusion practices, and our model can be used to study the general multi-unit exchange of indivisible objects with compatibility-based preferences over the objects, where for each agent both such preferences and her endowments are private information. To this end, we first reinterpret several elements in the model.

We consider I as a set of agents, and $_{i} \in B$ as the type of agent $i \in I$. For every $i \in I$, each $D_i \in D_i$ is a set of objects initially owned by agent i, i.e., the endowments of i, and $_{d} \in B$ is the type of each object $d \in D_i$. For every $X \in B$, there are v_X existing objects of type X that are not the endowments of any agent. We assume that

A mechanism f is strategy-proof if for any $i \in I$, $D, D' \in \mathcal{D}$, C and C' such that $D'_i \subseteq D_i$, $D_j = D'_j$, $C(\theta_j) = C'(\theta_j)$ for all $j \in I \setminus \{i\}$, and $f(D'|C')_X(i) = 0$ for every $X \in C'(\theta_i) \setminus C(\theta_i)$, we have

$$w_i\left(f(D|C)\right) R_i\, w_i\left(f(D'|C')\right):$$

Recall that, to incentivize an agent to report her full set of endowments, we require her feasible schedule set to become more favorable as she reports a larger set of endowments (Assumptions 3 and 4). Given that an agent may over-report or under-report her set of compatible types, we do not allow her feasible schedule set to vary with her preferences. That is, for each agent i, once D_i is given, $S_i(D_i)$ is fixed and does not depend on $C(\theta_i)$. Under the same assumptions on the feasible schedule correspondences as in Theorem 2, given an optimal mechanism, if an agent under-reports her endowment set and/or misreports her preferences, then she either receives an incompatible object, or receives weakly less compatible objects.

Theorem S.5. *Under Assumptions 1, 2 and 3, every optimal mechanism is weakly strategy-proof.*

Under these assumptions, the exchange rates in this general model can be endogenously determined k8r6be her compatible objects. This can be shown in the following two parts, because for an agent i and her two sets of compatible types $C(\theta_i)$ and $C'(\theta_i)$, we have $C'(\theta_i) = C(\theta_i) \cap B_1 \cup B_2$, where $B_1 = C(\theta_i) \cap C'(\theta_i)$ and $B_2 = C'(\theta_i) \cap C(\theta_i)$.

1. If any agent over-reports her set of compatible types, then she either receives an incompatible object, or receives weakly less compatible objects.
2. If any agent under-reports her set of compatible types, then she receives weakly less compatible objects.

cycle from M to M' is a directed graph of agents and objects in which each agent/object points to the next object/agent, and is denoted as a list $C = (i_1, d_1, \ldots, i_t, d_t)$, $t \geq 2$, such that for each $t \in \{1, \ldots, t\}$ (let $i_{t+1} = i_1$ and $d_0 = d_t$):

1. $i_t \in \hat{I}$, $d_t \in M'_{i_t} \setminus M_{i_t}$ and $d_t \in M_{i_{t+1}}$.
2. If $i_t \neq b$, $d_{t-1} \in D_{i_t}$, and $d_t \notin D_{i_t}$, then

$$(|M_{i_t} \setminus D_{i_t}| + 1, |D_{i_t} \setminus M_{i_t}| + 1) \in S_{i_t}(D_{i_t}) \quad \text{and} \quad (|M'_{i_t} \setminus D_{i_t}| - 1, |D_{i_t} \setminus M'_{i_t}| - 1) \in S_{i_t}(D_{i_t}).$$

3. If $i_t \neq b$, $d_{t-1} \notin D_{i_t}$, and $d_t \in D_{i_t}$, then

$$(|M_{i_t} \setminus D_{i_t}| - 1, |D_{i_t} \setminus M_{i_t}| - 1) \in S_{i_t}(D_{i_t}) \quad \text{and} \quad (|M'_{i_t} \setminus D_{i_t}| + 1, |D_{i_t} \setminus M'_{i_t}| + 1) \in S_{i_t}(D_{i_t}).$$

4. If i_t

Case 2. The procedure stops at i_t at Step $t-1$, $i_t \neq i_1$, and $|D_{i_1} \setminus M'_{i_1}| \leq |D_{i_1} \setminus M_{i_1}|$.

Then $(i_1; d_1; \ldots; d_{t-1}; i_t)$ is a chain from M to M'.

Case 3. The procedure stops at i_t at Step $t-1$, $i_t \neq i_1$, and $|D_{i_1} \setminus M'_{i_1}| > |D_{i_1} \setminus M_{i_1}|$. In this case, $(i_1; d_1; \ldots; d_{t-1}; i_t)$ may not be a chain from M to M'. We use the pointing procedure from M' to M, which starts with $j_1 = i_1$ pointing to some $c_1 \in D_{i_1}$ such that $c_1 \in M_{i_1} \setminus M'_{i_1}$. Then a cycle or a chain from M to M' can be found.

Case 4. The procedure stops at i_t at Step $t-1$ and $i_t = i_1$.

Subcase 4.1 $d_{t-1} \in D_{i_1}$.

To see that $(i_1; d_1; \ldots; i_{t-1}; d_{t-1})$ is a cycle from M to M', we verify Condition 2 in the definition of a cycle for i_1. Since $d_{t-1} \in D_{i_1}$ and $d_{t-1} \in M_{i_1}$, $|D_{i_1} \setminus M_{i_1}| < |D_{i_1}|$. Then given that $|M'_{i_1} \setminus D_{i_1}| > |M_{i_1} \setminus D_{i_1}|$, by Assumption 2, there exists $s > |D_{i_1} \setminus M_{i_1}|$ such that $(|M'_{i_1} \setminus D_{i_1}|; s) \in S_{i_1}(D_{i_1})$. It follows from Assumption 1' that

$$(|M_{i_1} \setminus D_{i_1}| + 1; |D_{i_1} \setminus M_{i_1}| + 1) \in S_{i_1}(D_{i_1}):$$

Similarly, $d_{t-1} \in D_{i_1}$ and $d_{t-1} \notin M'_{i_1}$ imply that $|D_{i_1} \setminus M'_{i_1}| > 0$. Then by Assumption 2, there exists $s' < |D_{i_1} \setminus M'_{i_1}|$ such that $(|M_{i_1} \setminus D_{i_1}|; s') \in S_{i_1}(D_{i_1})$. It follows from Assumption 1' that

$$(|M'_{i_1} \setminus D_{i_1}| - 1; |D_{i_1} \setminus M'_{i_1}| - 1) \in S_{i_1}(D_{i_1}):$$

Subcase 4.2 $d_{t-1} \notin D_{i_1}$ and $\succ_{d_{t-1}} \in \hat{C}'(\succ_{i_1})$.

Then $(i_1; d_1; \ldots; i_{t-1}; d_{t-1})$ is a cycle from M to M'.

Subcase 4.3 $d_{t-1} \notin D_{i_1}$ and $\succ_{d_{t-1}} \notin \hat{C}'(\succ_{i_1})$.

Then $(i_1; d_1; \ldots; i_{t-1}; d_{t-1})$ is not a cycle from $|$

a finite number of steps, some M^k, $k \geq 1$, is constructed and a cycle or a chain C from M^k to M^0 is found. Using arguments similar to those in the proof of Lemma 3, it can be shown that C is also a cycle or a chain from M to M^0. ■

Finally, by arguments similar to those in the proof of Lemma 4, we can use Lemma S.8 to show Lemma S.7. This concludes the proof of Theorem S.5.

C Weighted Maximal Mechanisms: Additional Results

C.1 Sequential Targeting Mechanisms are Weighted Maximal

Let $I = \{1, 2, \ldots, |I|\}$ be the set of patients. In this section, for the ease of matrix operations we use a slightly more general definition of an allocation. For every $D \in \mathcal{D}$, $\mu \in \mathcal{A}(D)$ and $i \in I$, $\mu_X(i)$ is defined for every blood type $X \in \mathcal{B}$ by setting $\mu_X(i) = 0$ for all $X \in \mathcal{B} \setminus \mathcal{C}(\tau_i)$.

Let f be a sequential targeting mechanism with respect to target sets $\{N_k\}_{k=1}^{\bar{k}}$ and target function σ. Consider any problem $D \in \mathcal{D}$. For each $k \in \{1, \ldots, \bar{k}\}$, we define a function $W_k : \mathcal{A}(D) \to \mathbb{Z}$ such that for every $\mu \in \mathcal{A}(D)$,

$$W_k(\mu) = \begin{cases} \sum_{i \in N_k, X \in \mathcal{B}} \mu_X(i) & \text{if } \sigma(k) = \max \\ \sum_{d \in \cup_{i \in N_k} D_i} \mu(d) & \text{if } \sigma(k) = \min \end{cases}.$$

Let $h \in \mathbb{Z}_{++}$. Define a function $W : \mathcal{A}(D) \to \mathbb{R}$ such that for every $\mu \in \mathcal{A}(D)$,

$$W(\mu) = \sum_{k=1}^{\bar{k}} h^{\bar{k}-k} W_k(\mu) = \sum_{i \in I} W^r(i)\,\mu(i) - W^s(i) \sum_{d \in D_i} \mu(d);$$

where

$$W^r(i) = \sum$$

if $\ell > k$, we have $W(\ ) > W(\ ')$ if

$$h^{k}\ {}^{k} > \sum_{\ell=k+1}^{K} h^{k-\ell} \sum_{X \in B} v_X + \sum_{i \in I} \max_{D_i' \in D_i} |D_i'| \ .$$

This is equivalent to

$$1 > \sum_{\ell=k+1}^{K} h^{k-\ell} \sum_{X \in B} v_X + \sum_{i \in I} \max_{D_i' \in D_i} |D_i'| \ .$$

Therefore, after choosing sufficiently large h such that

$$1 > \sum_{\ell=2}^{K} h^{1-\ell} \sum_{X \in B} v_X + \sum_{i \in I} \max_{D_i' \in D_i} |D_i'| \ ; \qquad (1)$$

we have for any $k \in \{1; \ldots; K\}$ and any $\ ;\ ' \in A_{k-1}$, $W_k(\ ) > W_k(\ ')$ implies $W(\ ) > W(\ ')$. Then, given that all the allocations in A_K are welfare equivalent, the sequential targeting outcome $f(D) \in A_K$ is welfare equivalent to any solution to the following maximization problem:

$$\max_{\ \in A(D)} W(\ )$$

Recall that each patient $i \in I$ first appears in a maximization target: for every $k \in \{2; \ldots; K\}$, if $(k) = \min$, then for any $i \in N_k$ there exists $k' < k$ such that $i \in N_{k'}$ and $(k') = \max$. This implies that for every $i \in I$, $W^r(i) \quad W^s(i)|D_i|$ for all $D_i \in D_i$, as h satisfies inequality (1). Therefore, f is a weighted maximal mechanism with respect to the score function with the individual weights $W^r(i)$ and $W^s(i)$. This shows the following the following maximization problem

$$\max_{2A(D)} W$$

Suppose that Assumption 1 (L-convexity) holds. Given 2 Z^a_+, we show that the constraint \ is an allocation", i.e., 2 $A(D)$, is equivalent to a system of linear inequalities in four parts:

1. For every patient i 2 I, let

$$r_i = \sum_{X 2B} {}_X(i) \quad \text{and} \quad s_i = \sum_{d2D_i} (d):$$

Since $S_i(D_i)$ is L-convex, there exists some integer vector b_i 2 Z^6 such that $(r_i; s_i)$ 2 $S_i(D_i)$ if and only if the following inequalities hold:

$$\begin{aligned} r_i \quad s_i \quad & b_{i;1} \\ r_i + s_i \quad & b_{i;2} \\ r_i \quad & b_{i;3} \\ r_i \quad & b_{i;4} \\ s_i \quad & b_{i;5} \\ s_i \quad & b_{i;6} \end{aligned}$$

We rewrite these linear inequalities in matrix form, after de ning

$$A_i = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 0 & 0 & 1 & 1 \end{pmatrix} \quad 8\, i \; 2 \; I; \qquad (2)$$

gr82e.173 Td [(C)]TJ 0 -7.1:[(C)-278793 Td [(()]TJ/F50 11.16701 T 12.619 0 Td [(b)]TJ

A gr82e.173 Td [(C)]TJ 0 -7.1:[(C)]3d [(=)]TJ/F56 11.9552 Tf 12.426 59.896.65 l12 -7.174 Td [(C)]TJ 0 - -7.173 Td [(C)][(I)

:9696d 25.115 7.97 Td [(.)]TJ 0 -3.985 Td [(.)]TJ 0 -3.985 Td [46

2. We rewrite the market clearing conditions,

$$\sum_{i\in I: X\in C(\ _i)} {}_X(i) \quad \sum_{d\in\bigcup_{i\in I} D_i:\ {}_d = X} (d) \le v_X \qquad \forall X \in B;$$

in matrix inequality form as

$$A_B \le v \tag{4}$$

where

$$A_B = (A_X^T)_{X\in B}$$

defined by $\forall X \in B$,

$$A_X = \left(A_X(i;Y)_{Y\in B};\ A_X(d)_{d\in D_i} \right)_{i\in I}$$

such that

$$A_X(i;Y) = \begin{cases} 1 & \text{if } Y = X \text{ and } X \in C(\ _i) \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I;\ \forall Y \in B$$

and

$$A_X(d) = \begin{cases} 1 & \text{if } {}_d = X \\ 0 & \text{otherwise} \end{cases} \qquad \forall d \in \bigcup_{i\in I} D_i:$$

3. The following inequality states that a donor never exceeds 1 unit of donation:

$$(d) \le 1 \qquad \forall d \in \bigcup_{i\in I} D_i:$$

We rewrite this as

$$A_D \le b_D = (1;\dots;1) \tag{5}$$

where

$$A_D = \left(A_D(r;c) \right)_{r \quad a;\ c\ \in \bigcup_{i\in I} |D_i|}$$

such that $A_D(r;c) = 1$ if both row r and column c refer to the same donor d, and $A_D(r;c) = 0$ otherwise.

4. Finally, no patient receives incompatible blood:

$$\sum_{i\in I} \sum_{X\in B\setminus C(\ _i)} {}_X(i) \le 0;$$

which can be written as

$$A_C \le 0 \tag{6}$$

where

$$A_C = \left(\left(A_C(i;X)_{X\in B};\ A_C(d)_{d\in D_i} \right)_{i\in I} \right)^T$$

such that

$$A_C(i;X) = \begin{cases} 1 & \text{if } X \notin C(\ _i) \\ 0 & \text{otherwise} \end{cases} \qquad \forall i \in I;\ \forall X \in B$$

and

$$A_C(d) = 0 \quad \forall d \in \cup_{i \in I} D_i .$$

Then the vector $\in \mathbb{Z}^a_+$ is an allocation, i.e., $\in \mathcal{A}(D)$, if and only if inequalities (3), (4), (5), and (6) hold. This implies that the following integer linear program in cannonical form finds an allocation that is welfare equivalent to $f(D)$:

$$\max \quad W \tag{7}$$

subject to

$$A \quad b \tag{8}$$

where

$$A = (A_I; A_B; A_D; A_C) \quad \text{and} \quad b = (b_I; v; b_D; 0)$$

such that is a $1 \quad a$ non-negative integer vector, A is an $a \quad (6|I| + |B| + |\cup_{i \in I} D_i| + 1)$ integer matrix with entries 0, 1 or 1, and b is a $1 \quad (6|I| + |B| + |\cup_{i \in I} D_i| + 1)$ integer vector. We consider its linear program relaxation such that the search space is $\mathbb{R}^a_+$ instead of $\mathbb{Z}^a_+$.

A matrix is totally unimodular if the determinant of every square submatrix is $1; 0$ or 1. The following result is well known and straightforward to prove using Cramer's rule in linear algebra (for example, see Schrijver (1998)).

Lemma S.9. The vertices of the polyhedron defined by the inequality (8) are integer-valued for any integer vector b if and only if A is totally unimodular.

Thus, for any linearly independent basis for the linear program relaxation of the problem in (7) and (8) has only integer solutions for any integer vector b if and only if A is totally unimodular. The following lemma establishes a condition for checking the total unimodularity of A:

Lemma S.10 (Ghouila-Houri (1962)). A is totally unimodular if and only if there exists a partition of any subset of column indices $C \quad \{1; 2; \ldots; 6|I| + |B| + |\cup_{i \in I} D_i| + 1\}$ as K_C and L_C such that for the column vector $= \sum_{c \in K_C} A^c \quad \sum_{c \in L_C} A^c$, where A^c is the c^{th} column vector of A, we have $(r) \in \{ 1; 0; 1\}$ for every row $r = 1; \ldots; a$.

We prove that A is indeed totally unimodular using this result.

Lemma S.11. The matrix A is totally unimodular.

Proof of Lemma S.11. Let $C \quad \{1; 2; \ldots; 6|I| + |B| + |\cup_{i \in I} D_i| + 1\}$ be any subset of column indices of A. We construct a partition of C, K_C and L_C, as in Lemma S.10 in four steps. Below for each $x \in \{1, 2, 3, 4\}$, ^{x} denotes the difference vector between the sum of the columns with indices in K_C and the sum of the columns with indices in L_C at the end of the construction in Step x.

1. We first consider the columns that correspond to the feasible schedule constraints. Let $i \in I$. List the column indices in the set $\{c \in C : 6(i - 1) + 1 \leq c \leq 6i\}$ as $c_1, c_2, \ldots, c_k$. We will inductively assign these indices to two sets, K_C^i and L_C^i, which are both initialized to $\emptyset$. Let the index of the first row regarding i in each column be

$$r = (i - 1)|B| + \sum_{j<i} |D_j| + 1;$$

- If exactly one of (1) and (1) is 0 and exactly one of (2) and (2) is 0: Then suppose (m) and (n) are nonzero for $m \neq n$. If they have the same sign, then assign c_ℓ to L^i_C. If they have opposite signs, then assign c_ℓ to K^i_C. Thus,

$$(\ \ell(r);\ \ell(r'))=(\ \ x;x)$$

where $x \in \{\ \ 1;1\}$.
- If (1) and (1) have the same sign, then assign c_ℓ to L^i_C; and if they have opposite signs, then assign c_ℓ to K^i_C. In the former case (2) and

On the other hand, for any row r' regarding a donor,

$$\sum_{\ell=1}^{k} A(r', c_\ell) \in \{-1, 0\}.$$

Assign every c_ℓ to L_C. Then we have

$$\delta^2 = \delta^1 - \sum_{\ell=1}^{k} A^{c_\ell};$$

which is the difference vector between the sum of the columns with indices in K_C and the sum of the columns with indices in L_C at the end of Step 2.

For the row r defined above we have $\delta^2(r) \in \{-1, 0, 1\}$ as $\delta^1(r) \in \{0, 1\}$. For the row r' defined above we have $\delta^2(r') \in \{-1, 0, 1\}$ as $\delta^1(r) \in \{-1, 0\}$.

3. For any $c \in C$ with $6|I| + |B| < c < 6|I| + |B| + |\cup_{i \in I} D_i| + 1$, column c is in A_D and refers to some donor with a row number r. Assign c to L_C if $\delta^2(r) \in \{0, 1\}$, and assign it to K_C otherwise. After all such column indices are assigned, $\delta^3(r) \in \{-1, 0, 1\}$ for any row r regarding a donor, and $\delta^3(r') = \delta^2(r') \in \{-1, 0, 1\}$ for any other row r'.
4. The last column of A is the vector A_C and $A_C(r) \in \{0, 1\}$ for every row r. Consider any row r such that $A_C(r) = 1$. This refers to a patient i and a blood type X such that $X \notin \mathcal{C}({}_i)$. Then for any $c \in C$ assigned in Steps 2 and 3, $A(r; c) = 0$. Therefore, $\delta^3(r) = \delta^1(r) \in \{0, 1\}$. If the index $6|I| + |B| + |\cup_{i \in I} D_i| + 1 \in C$, we assign it to L_C so that $\delta^4(r) \in \{-1, 0\}$. For any row r' such that $A_C(r') = 0$, $\delta^4(r') = \delta^3(r') \in \{-1, 0, 1\}$.

Therefore, we have constructed a partition of C, K_C and L_C, such that $\sum_{c \in K_C} A^c - \sum_{c \in L_C} A^c = \delta^4$ and $\delta^4(r) \in \{-1, 0, 1\}$ for every row $r = 1, \ldots, a$. By Lemma S.10, A is totally unimodular. ■

These results are used to prove the following proposition.

Proposition 2. Under Assumption 1, the outcome of a weighted maximal mechanism can be found in polynomial time.

Proof of Proposition 2. By Lemmata S.9 and S.11, under Assumption 1, all the basic solutions to the linear program relaxation of the integer linear program in (7) with constraint (8) are integer-valued. Thus, any polynomial LP method, such as the simplex algorithm, finds an allocation that is welfare equivalent to $f(D)$ in polynomial time. ■

D Examples Regarding Violations of Assumptions

Example S.3 and Example S.4 below show that Assumption 1 and Assumption 2 are needed for the donor monotonicity of the optimal mechanisms, respectively.

Example S.3 (Violation of Assumption 1). Suppose that the set of patients is $I = \{1; 2; 3; 4\}$. For every $i \in I$, $\underline{n}_i = 0$. Each patient's blood type, maximum need and donor set are given as follows.

- $_1$ = A, $\bar{n}_1 = 2$, and Patient 1 has two type B donors and four type O donors.
- $_2$ = B, $\bar{n}_2 = 2$, and Patient 2 has four type O donors.
- $_3$ = O, $\bar{n}_3 = 4$, and Patient 3 has one type A donor and seven type AB donors.
- $_4$ = A, $\bar{n}_4 = 1$, and Patient 4 has two type AB donors.

In addition, the blood bank only has one unit of type A blood in its inventory. Assume ABO-identical transfusion.

For every $i \in I$ and every possible donor set $D_i \in \mathcal{D}_i$,

$$S_i(D_i) = \left\{ (r; s) \in W_i : s = 2r \text{ and } r \leq \min \left\{ \bar{n}_i; \; |D_i| = 2 \right\} \right\}:$$

Note that Assumptions 2 and 3 are satisfied, while Assumption 1 is violated: if a patient reports at least two donors, then her feasible schedule set is not L-convex.

Let f be a sequential targeting mechanism with respect to target sets $\{N_k\}_{k=1}^{K}$ and target function such that $N_1 = N_2 = \{3\}$

- $_3 = AB$, and Patient 3 has one type A donor and one type O donor.
- $_4 = O$, and Patient 4 has one type A donor.

In addition, the blood bank only has one unit of type AB blood in its inventory. Assume ABO-identical transfusion.

The exchange rate is one-for-one for every $i \in I \setminus \{1\}$. That is, for every reported donor set $D_i \in \mathcal{D}_i$, where $i \in I \setminus \{1\}$,

$$S_i(D_i) = \begin{cases} \{(0;0)\} & \text{if } D_i = \emptyset \\ (0;0);(1;1) & \text{otherwise} \end{cases} :$$

On the other hand, Patient 1 can receive blood up to her maximum need by supplying at most one unit: for every $D_1 \in \mathcal{D}_1$,

$$S_1(D_1) = \begin{cases} \{(0;0)\} & \text{if } D_1 = \emptyset \\ (0;0);(1;0);(1;1);(2;0);(2;1) & \text{otherwise} \end{cases} :$$

This is a special case of the Delhi policy in Example 1. Note that Assumptions 1 and 3 are satisfied. However, Assumption 2 is violated, since when Patient 1 reports two donors, $(2;2)$ is not a feasible schedule.

Let f be a sequential targeting mechanism with respect to target sets $\{N_k\}_{k=1}^{k}$ and target function such that $N_1 = \{2\}$. Then f selects the following allocation when every patient truthfully reports her donor set:

- Each $i \in I$ receives one unit of type $_i$ blood.[70]
- Patient 1's type B donor donates, Patient 3's type O donor donates, and the donor of $i \in \{2;4\}$ donates.

If Patient 1 conceals her type B donor, then

It can have preferences over different remaining inventories and such preferences can correspond to some explicit objectives, such as maximizing the amount of certain types of blood in stock. To this end, we extend our model and include the blood bank b as an agent. In an allocation μ, we also specify the amount of type X blood the bank receives, $\mu_X(b)$, for each $X \in \mathcal{B}$. Denote a blood bundle that the bank keeps in its inventory as $z = (z_X)_{X \in \mathcal{B}} \in \mathbb{Z}_+^{|\mathcal{B}|}$. Assume that the bank has a complete preference relation over all the blood bundles. Then the definition of efficiency can be modified accordingly to include the bank's welfare. A schedule profile is extended and denoted by a vector $w = \left((r_i; s_i)_{i \in I}; (z_X)_{X \in \mathcal{B}}\right) \in \mathcal{W} \times \mathbb{Z}_+^{|\mathcal{B}|}$. The mechanism designer's preference relation $\succeq$ over all such schedule profiles is complete, transitive, antisymmetric, and responsive to the basic schedule profiles in the set $\{0; 1\}^{2|I| + |\mathcal{B}|}$. Moreover, $\succeq$ is aligned with the preferences of all the agents (all the patients and the bank): for every two schedule profiles w and w', we have $w \succ w'$ if every agent weakly prefers w to w', and at least one agent strictly prefers w to w'.[71] Then, the optimal mechanism induced by $\succeq$ is efficient, and it is straightforward to extend the proofs to show that Theorem 2 and Theorem 3 remain valid.

We give a simple and concrete example of an optimal mechanism in this more general with every agent's preferences. In particular, the specification of the target sets N_k and N_ℓ ensures that it is aligned with the bank's preferences.[72]

E.2 Integrated Blood Component Markets

Although in practice replacement donor programs function for each blood component separately, it is plausible that higher welfare gains can be achieved by integrating these markets. For instance, a patient requesting red blood cells can have her donors donate platelets to another patient, while the latter patient's donors donate red blood cells to the

the type of each patient i, $_{i}$, is extended to specify which component she needs. Hence,

$$B' = \{(c, X) : c \in \{rbc, plt, wb\} \text{ and } X \in \{O+, O-, A+, A-, B+, B-, AB+, AB-\}\}$$

is the set of patient types. We assume that each donor can donate either one unit of apheresis platelets, or one unit of whole blood, which can simply be used as a whole blood transfusion pack, or to prepare one unit of red blood cells. Therefore, each donor d can provide 1 unit of