Allocation Mechanisms Without Reduction

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Abstract

We study a simple variant of the house allocation problem (one-si

outcomes, but typically not in the way these procedures are perceived by the agents to whom outcomes are allocated. The literature on allocation problems, and more generally on one-sided and two-sided matching (for a survey, see Abdulkadiroglu and Sönmez (2013)), typically maintains the assumption that agents are only interested in the overall probability they will receive their desired outcome. This assumption implies the equivalence of different randomized mechanisms (Abdulkadiroglu and Sönmez (1998); see also Pathak and Sethuraman (2011)). Nevertheless, we show that taking into account the procedures that generate this probability may be important, and seemingly similar mechanisms like Random Top Cycle and Random Serial Dictatorship can be ranked differently when preferences over compound lotteries are taken into consideration. Moreover, we use this insight to propose a new mechanism which may be better than those currently discussed in the literature.

The basic structure we investigate is simple: N units of two different types need to be allocated to N individuals, one per person. For example, N dorms — some face west and the other face east — that need to be allocated to incoming students. Some will prefer one type and some will prefer the other. Since there are only two types, there is no room for strategic manipulations and agents' optimal strategy is to reveal their true preferences. For tractability, in the formal analysis we will confine attention to the case of a large population: a continuum of agents and units.

We first take an ex-ante approach, where agents are yet to learn their own preferences (as well as those of the other agents) over the goods. These preferences are revealed after the first part of the procedure takes place. In professional sports, for example, teams typically know their rank in the draft before they know which positions they would like to fill. This will become clearer by the time they know the draft prospects and the medical condition of their current roster for next year. In a school context, prospective students often attend visit days and open houses long after the assignment procedure has been announced. Individuals in such situations thus view possible mechanisms as compound lotteries, that is, lotteries over the interim probabilities of receiving their desired outcome. Crucially, our analysis is based on the assumption that sequential probabilities are not taken by individuals to be the same as their product. In other words, individuals do not obey the reduction of compound lotteries axiom, according to which an agent should be indifferent between any multi-stage lottery and the single-stage lottery that induces the same probability distribution over final outcomes.

Extensive experimental results suggest that individuals often fail to reduce compound lotteries to simple ones using probability laws (see, among others, Halevy (2007), Abdellaoui, Klibanoff, and Placido (2015), Harrison, industries, for example, in the airline industry. While priority groups often depend on some merit (e.g. the amount spent and miles flown in the previous calendar year), in the alternative we suggest the assignment to groups is random.

Intuitively, the different mechanisms we consider induce lotteries over interim probabilities that can be ranked in terms of "riskiness." Following Machina (1982), we approximate local behavior by expected utility functionals, and the curvatures of these local utility functions determine the desirability of a mechanism. For example, TC generates a less risky distribution than SD, and thus is preferred if all local utilities are concave. We outline conditions over these utilities that determine preferences over mechanisms and show that these conditions can also be linked to attitudes towards the timing of resolution of uncertainty (see section IV).

While our analysis is mainly focused on the ex-ante approach, it can also be applied to the interim case, in which each individual learns his place in the mechanism after he already knows his type. We demonstrate this in Section VI. Here again, the absence of the reduction assumption allows us to examine the performance of seemingly identical mechanisms, and to show conditions under which TC or SD is superior to the other and conditions where PG is preferred to both. More generally, it is important to emphasize that our analysis is not restricted to any specific order. It applies whenever mechanisms involve some sequential resolution of uncertainty (even with more than two stages) and individuals do not obey the reduction of compound lotteries assumption.

The rest of the paper is as follows: Section II introduces the basic structure and the TC and SD mechanisms. Section III describes the preferences we consider and compares TC to SD. Section IV discusses the PG mechanism and provides conditions under which it is preferred to both TC and SD. Section V extends our analysis beyond the class of preferences we studied in the previous sections. Section VI considers the interim case. Section VII concludes with some further discussion.

II Two Allocation Mechanisms

Consider the following continuum variant of the house allocation problem (Hylland and Zeckhauser (1979)). There are goods of two types, g_1 and g_2 in proportion $p : 1 - p$, to exactly supply the total quantity needed to accommodate a $[0,1]$ continuum of agents. All agents have the same stochastic preferences, where with probability q each prefers g_1 to g_2 (independently of the preferences of others). We normalize payoffs so that the utility from the desired outcome is 1 and the utility from the other outcome is 0. We analyze below the case of excess supply of g_1, that is, $p > q$. The analysis of the case $p < q$ is similar.

In this section we consider two familiar mechanisms, each consisting of two stages.

> Random Top Cycle (TC): In the first stage, the goods are randomly allocated among the agents, so that the probability of person i holding good of type g_1 or g_2 is p or $1 - p$, respectively. In the second stage, the entire profile of preferences is revealed and trade, if needed, takes place. Those who like their holding will keep it. The rest will trade according to the following schedule: If m proportion of people holding one type of good and $\ell < m$ proportion of people holding the other type are unhappy with their holding, then the latter group will trade and get their desired outcome, while ℓ out of the former group will be selected at random and get their preferred option. The other $m - \ell$ will keep their undesired outcome.[2]
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> Random Serial Dictatorship (SD): In the first stage the order of the agents is randomly determined, so that the probability of each person being in the top m part of the queue is m. In the second stage, the entire profile of preferences is revealed. The agents then choose goods according to the order determined in the first stage. Agents get their desired outcome if, when their turn arrives, such a unit is still available.

and SD lead to the same overall probability of success, and are hence deemed indifferent if agents are only interested in the overall probability at which they will receive their desired outcome (see Abdulkadiroglu and Sönmez (1998)).

Suppose that the individual knows that he will face a binary lottery of the form $(x;\ ;y;1\ )$ where x and y are fixed and x is preferred to y, but the winning probability is determined by a random device such that with probability $_i$ the value of is $_i$. We denote such lotteries as $\langle {}_1; {}_1; \ldots; {}_n; {}_n\rangle$. This is a two-stage lottery, where in the first stage, with probability $_i$ the winning probability of the second stage is determined to be $_i$. The second stage is a simple lottery over the final outcomes x and y, where the former is obtained with probability $_i$.

Consider first the TC mechanism. Since there is an excess supply of g_1 ($p > q$), all those who receive g_2 | their proportion is $1\ p$ of the population | know that they will end up with their desired outcome regardless of their preferences. Either they will like it and keep it, or they will be able to trade. The size of the group of those who will receive g_2 but would like to replace to satisfy their desires only if they prefer g_1 to g_2. The probability of having these preferences is q. SD thus leads to the lottery over probabilities given by $X_2 = \langle 1, \frac{1-p}{1-q}; q, \frac{p-q}{1-q} \rangle$. Here too the analysis is ex-ante, before individuals know their position in the queue or their preferences (which will only be revealed to them later). Observe that if $p = q$, then in a large economy both TC and SD yield (almost) everyone his desired outcome for sure.

III Are TC and SD Equivalent?

The two lotteries over the probabilities of success we have previously discussed, $X_1 = \langle 1, 1-p; \frac{q}{p}, p \rangle$ and $X_2 = \langle 1, \frac{1-p}{1-q}; q, \frac{p-q}{1-q} \rangle$, have the same "expected value." That is, the expected probability of receiving the preferred good is the same under both mechanisms, which is $1-p+q$. This is not surprising. As p, the proportion of good g_1, is greater than q, the proportional demand for g_1, it must be that eventually $p-q$ agents will not be happy with their outcome. Ex-ante, when agents do not yet know their preferences and the outcome of the mechanism, the reduced probability of success for each of them is therefore $1-p+q$. This, however, does not necessarily mean that all mechanisms with this reduced probability are equally attractive.

Let x = "receive the desired outcome" and y = "receive the undesired outcome." As x and y are fixed, the probability ${}_i$ represents the lottery $(x, {}_i; y, 1-{}_i)$. The decision maker has preferences over compound lotteries of the form $\langle {}_1, {}_1; \ldots; {}_n, {}_n \rangle$ which can be represented by a functional V. Following Kreps and Porteus (1978) and Segal (1990), we use the recursive analysis of preferences over compound lotteries, where the decision maker considers the two-stage lottery $\langle {}_1, {}_1; \ldots; {}_n, {}_n \rangle$ as a lottery over his subjective values of the lotteries $(x, {}_i; y, 1-{}_i)$. In particular, we do not assume the reduction of compound lotteries axiom, hence V is not ordinally equivalent to $\sum {}_i {}_i$.

We analyze mechanisms as cumulative distribution functions over $[0, 1]$, where $F_X(\ )$ is the probability that the mechanism X yields a simple lottery $(x, \ ; y, 1-\ )$ with them, where at least one of these improvements is strict. Ex-post efficiency implies that every individual who prefers the good for which there is excess supply *must* obtain it, as otherwise there will be scope for an improving trade. Therefore, any lottery in which some fraction of the population know for sure that independent of their preferences they will not receive their desired outcome (that is, any lottery over ex-ante probabilities in which $_i = 0$ is in its support) will be inefficient and will not be considered a valid mechanism. On the other hand, both TC and SD are ex-post efficient. It is enough to show that there is no agent who holds the item for which there is excess demand while he prefers the other good. By the construction of the TC mechanism, any such individual will participate in the second stage trade. In SD, such an individual will never choose this good when his turn arrives, as his preferred good, which is in excess supply, will be still available.

Following Machina (1982), we assume first that the representation function V is smooth in the sense of being Frechet differentiable: For every F there exists a continuous *local utility* function $u_F(\ )$ over $[0;1]$ such that

$$V(G) \quad V(F) = \int_0^1 u_F(\ )\,d(G(\ )\,G($$

Risk aversion along a segment connecting two lotteries may be a reasonable assumption when lotteries are over monetary payoffs, but such an attitude is much less obvious in the present context. To illustrate, consider lotteries of the form $Y_\varepsilon = \langle \frac{1}{2} - \varepsilon; \frac{1}{2}; \frac{1}{2} + \varepsilon; \frac{1}{2} \rangle$. Obviously they are ordered by mean-preserving spread, where for $\varepsilon' > \varepsilon$, $Y_{\varepsilon'}$ is a mean preserving spread of Y_ε. However, there is no obvious reason to posit a specific ranking between Y_0 and $Y_{0.5}$. If time is not involved, it seems plausible to assume that $Y_0 \sim Y_{0.5}$, as both represent a simple even chance of winning. If the passage of real time is considered, then preferences between the two capture preferences over the timing of resolution of uncertainty, as $Y_{0.5}$ is fully resolved in the current period, whereas Y_0 only resolves later. As we will further discuss in Section IV, there is no empirically obvious pattern for such preferences. We are thus interested also in situations where the local utilities are not always concave or always convex. We use this in the next section where we offer a new mechanism and show conditions under which this mechanism is better than both TC and SD.

IV The Priority Groups Mechanism

In this section we offer an alternative new mechanism, called *Priority Groups* (PG), and provide conditions under which it is preferred to both TC and SD. This mechanism first allocates the two goods as in the TC mechanism, and those who will be assigned to the top priority group will obtain their desired outcome even if they received g_1. The size of this set is pr_1. As the allocation

Let

$$A(r_1) = q + \frac{(1-p)q - p(1-q)r_1}{p(1-r_1)}$$

Eq. (3) becomes

$$X_3(r_1) = \langle 1; 1-p+pr_1; A(r_1); p(1-r_1)\rangle \tag{5}$$

Observe that if $r_1 = 0$, then $X_3(r_1)$ reduces to $X_1 = \langle 1; 1-p; \frac{q}{p}; p\rangle$, the lottery obtained by the TC mechanism. On the other hand, if $r_1 = \frac{q(1-p)}{p(1-q)}$, then by eq. (4) $s_2 = 0$ and $X_3(r_1)$ reduces to $X_2 = \langle 1; \frac{1-p}{1-q}; q; \frac{p-q}{1-q}\rangle$, the lottery obtained by the SD mechanism.

We now show that under some simple conditions, neither TC nor SD are optimal. We will do this by showing that moving from either in the direction of the PG mechanism will make individuals better off ex-ante.

Denote by u_{r_1} be the local utility $u_{X_3(r_1)}$ of V at $X_3(r_1)$.

Proposition 2 If u_0 is convex on $[\frac{q}{p}; 1]$ and $u_{\frac{q(1-p)}{p(1-q)}}$ is concave on $[q; 1]$, then neither TC nor SD is optimal.

Proof: By Machina's (1982) analysis

$$\begin{aligned}\frac{\partial}{\partial r_1}V(X_3(r_1)) &= \frac{\partial}{\partial r_1}\mathrm{E}[u_{r_1}(X_3(r_1))] \\ &= pu_{r_1}(1) - \frac{p-q}{1-r_1}u'_{r_1}(A(r_1)) - pu_{r_1}(A(r_1))\end{aligned}$$

As $A(0) = \frac{q}{p}$, we get

$$\frac{\partial}{\partial r_1}V(X_3(r_1))\Big|_{r_1=0} = pu_0(1) \; (($$

For the second part, denote $r_1 = \frac{q(1 \quad p)}{p(1 \quad q)}$. As $A(r_1) = q$, we get

$$\frac{@}{@r_1} V(X_3(r_1)) \Big|_{r_1=r_1} = pu_{r_1}(1) \quad p(1 \quad q)u'_{r_1}(q) \quad pu_{r_1}(q)$$

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$_{(1 \quad q)}[u_{r_1}(1) \quad 7 \quad (r)]1 \; 300.050$

example, quadratic utility (Chew, Epstein, and Segal (1991)) and weighted expected utility (Chew (1983))), there are many popular models that are not. A prominent example is rank-dependent utility (Quiggin (1982)).[4] Our aim is to show that the possible improvement of the PG mechanism over both TC and SD holds more generally. We demonstrate this by providing sufficient conditions on a version of the rank-dependent utility functional known as the dual theory (Yaari (1987)).[5]

Since $1 > A(r_1)$, the rank-dependent value of $X_3(r_1)$ (see eq. (5)) is

$$V_{\mathrm{RD}}(X_3(r_1)) = u(A(r_1))g(p(1 \quad r_1)) + u(1)[1 \quad g(p(1 \quad r_1))]$$

By the definition of $A(r_1)$ we get

$$\begin{aligned} \frac{\partial}{\partial r_1}V_{\mathrm{RD}}(X_3(r_1)) &= \frac{\partial}{\partial r_1}[u(A(r_1))g(p(1 \quad r_1)) + u(1)[1 \quad g(p(1 \quad r_1))]] = \\ &\quad \tfrac{p \quad q}{p(1 \quad r_1)^2} \quad u'(A(r_1))g(p(1 \quad r_1)) \\ &\quad pu(A(r_1))g'(p(1 \quad r_1)) + pu(1)g'(p(1 \quad r_1)) \end{aligned} \tag{6}$$

Recall that $r_1 = 0$ represents the TC case. We get

$$\frac{\partial}{\partial r_1}V_{\mathrm{RD}}(X_3(r_1))\Big|_{r_1=0} = \frac{p \quad q}{p}u' \quad \frac{q}{p} \quad g(p) \quad pg'(p) \;\; u \quad \frac{q}{p} \qquad u(1)$$

For example, for $u(\ \ ) = \ \ $ this equation becomes

$$\frac{p \quad q}{p}g(p) \quad p \;\; \frac{q \quad p}{p} \;\; g'(p)$$

which is positive if and only if

( ) $\ {}_g(p) > 1$

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[4] Other examples include Gul (1991) and Cerreia-Vioglio, Dillenberger, and Ortoleva (2015).

[5] If we order the prizes in the support of a lottery $\langle\ {}_1,\ {}_1, \ldots,\ {}_n,\ {}_n\rangle$, with ${}_1 > {}_2 > \ldots > {}_n$, then the functional form for rank-dependent utility is: $V(\langle\ {}_1,\ {}_1, \ldots,\ {}_n,\ {}_n\rangle) = u(\ {}_n)g(\ {}_n) + \sum_{i=1}^{n\ 1} u(\ {}_i)[g(\sum_{j=i}^{n}\ {}_j) \quad g(\sum_{j=i+1}^{n}\ {}_j)]$, where $g : [0,1] \to [0,1]$ is strictly increasing and onto, and $u : [0,1] \to \mathrm{R}$

The SD case is obtained when $r_1 = \frac{q(1\quad p)}{p(1\quad q)}$. Now

$$\frac{\partial}{\partial r_1}V_{\mathrm{RD}}(X_3(r_1)) \quad {}_{r_1=\frac{q(1\quad p)}{p(1\quad q)}} =$$

$$\frac{p(1\quad q)^2}{p\quad q}u'(q)\,g\quad\frac{p\quad q}{1\quad q}\qquad pg'\quad\frac{p\quad q}{1\quad q}\quad[u(q)\qquad u(1)]$$

which is negative if and only if

$$\frac{u'(q)(1\quad q)}{1\quad u(q)} > \quad {}_{g}\quad\frac{p\quad q}{1\quad q}$$

For $u(\ ) =$ this condition becomes

$$(\quad)\ 1 > \quad {}_{g}\quad\frac{p\quad q}{1\quad q}$$

It is common to assume that g is an inverse S-shaped function — concave for small probabilities and convex for high probabilities.[6] This property

If g is strictly increasing, there is a unique r_1 with this property. Note that this condition is consistent with ( ) and ( ) since $p > p(1-r) > \frac{p-q}{1-q}$ (recall that $r_1 \in \left[0, \frac{q(1-p)}{p(1-q)}\right)$). Differentiate again to obtain

$$\frac{p-q}{1-r_1}\frac{2g(p(1-r_1))}{p(1-r_1)^2} - \frac{2g'(p(1-r_1))}{1-r_1} + g''(p(1-r_1)) \tag{8}$$

Using eq. (7), the sign of expression (8) is the same as the sign of $-g''(p(1-r_1))$. It is thus negative if $g''(p(1-r_1)) > 0$, in which case r_1 is indeed optimal.

VI Known Preferences

In previous sections we studied the ex-ante case, where the first part of the mechanism is implemented before individuals know their own preferences. A similar method can be used to study the case where individuals know their preferences from the beginning. Here we demonstrate how our analysis can be easily applied to this case as well. As before, we confine attention to the case of large (continuum) economies and assume, without loss of generality, that there is an excess supply of g_1, that is, $p > q$. In the TC mechanism we can therefore identify four groups:

1. qp will get g_1 and like it.
2. $(1-q)p$ will get g_1 and will prefer to trade it for g_2.
3. $q(1-p)$ will get g_2 and will prefer to trade it for g_1.
4. $(1-q)(1-p)$ will get g_2 and like it.

Since $p > q$, the third group is smaller than the second one, and therefore all members of the third group will be able to trade. In other words, all those who prefer g_1 (the first and the third group) are guaranteed to receive it. Those who prefer g_2 face a lottery. With probability $1-p$ they will get their desired outcome, and with probability p they will get their desired outcome if they will be able to trade, the probability of this event is $\frac{q(1-p)}{(1-q)p}$.)87.86

where $(1 \quad q) \quad = 1 \quad p$, that is, if his rank is less than $\frac{1 \ p}{1 \ q}$. The underlying conditional lottery is thus $W_2 = h1; \frac{1 \ p}{1 \ q}; 0; \frac{p \ q}{1 \ q} i$. Note that here, after the first step of the mechanism, all participants know for sure whether they will

away from any strategic considerations by confining attention to a setting in which there are only two types of goods, and use individual preferences over mechanisms to compare them. We thus concentrate on the decision-theoretic dimension of the mechanisms without having to worry about individuals' manipulations of their preferences over the outcomes.

Random allocation mechanisms typically involve multi-stage lotteries. Based on a compelling evidence that people do not routinely use the laws of probability to reduce multi-stage lotteries, we postulate that individuals perceive mechanisms as compound lotteries and have recursive preferences over them. Simple and familiar conditions then allow us to compare mechanisms that are deemed identical in standard models. Moreover, our approach permits us to offer a new mechanism that under some conditions outperforms standard mechanisms.

In this paper we show that it is enough to have $n = 2$ priority groups to (sometimes) improve upon both TC and SD. In the special case of rank-dependent utility we also outline conditions for an optimal PG mechanism with two groups. We do leave open, however, the question of what is the optimal number of groups (together with the probability of trade assigned to each of them). This question crucially depends on the individuals' preferences.

There are some considerations about the actual implementation of the PG mechanism which we do not explicitly address in the paper. First, it is often the case that having more groups entails higher bureaucratic costs that may offset the benefit of having finer division of the population. Therefore, even if it is theoretically beneficial to have more priority groups, a cost-benefit analysis may dictate a smaller number of such groups.

Implementation of the extreme cases of TC and SD requires no knowledge of individual preferences or even aggregate preferences. The goods are allocated at random (TC) and individuals are ordered at random (SD) regardless of preferences. In order to determine which is better, however, society needs to have information about the value of q and about individual preferences over lotteries.

The allocation of the goods in the PG mechanism does not require knowledge about q, but the determination of the sizes of the groups and their probabilities of trade needs to satisfy eq. (2) which requires q.[7] Moreover, as is

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[7] For the analysis in Section V, knowing q is needed only to the extent that it determines the possible range of r_1, the size of the group that is guaranteed the option to trade.

demonstrated in Section V, the optimal division into priority groups requires knowledge about individual preferences.

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